

Rigorous relativistic equation for quark-antiquark bound states at finite temperature derived from the thermal QCD formulated in the coherent-state representation

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A rigorous three-dimensional relativistic equation for quark-antiquark bound states at finite temperature is derived from the thermal QCD generating functional which is formulated in the coherent-state representation. The generating functional is derived newly and given a correct path-integral expression. The perturbative expansion of the generating functional is specifically given by means of the stationary-phase method. Especially, the interaction kernel in the three-dimensional equation is derived by virtue of the equations of motion satisfied by some quark-antiquark Green functions and given a closed form which is expressed in terms of only a few types of Green functions. This kernel is much suitable to use for exploring the deconfinement of quarks. To demonstrate the applicability of the equation derived, the one-gluon exchange kernel is derived and described in detail.

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I. INTRODUCTION

Quantum Chromodynamics (QCD), as a strong interaction theory of quarks and gluons, has a distinctive property of asymptotic freedom and infrared slavery which makes the quarks and gluons to be confined in hadrons. It is widely believed that in extreme conditions, i.e., at high temperature and/or high density, the quarks and gluons would be deconfined from hadrons and form a new matter, the quark-gluon plasma (QGP). It is highly expected that the QCD phase transition from hadrons to QGP would take place and be observed in the high energy heavy ion collisions at RHIC [1-3]. Theoretically, to predict the QCD phase transition, many efforts have been made by using different approaches such as the lattice simulation, the effective field theory, the hydrodynamic model and etc. [4-14]. According to the prediction of the lattice QCD calculations, the phase transition could occur when the colliding system reaches the temperatures 150-170 MeV [2]. Since the quarks and gluons are confined in hadrons which exist as the bound states of quarks and/or gluons, it is obvious that a proper approach of investigating the quark deconfinement is to start from an exact relativistic equation for quark and/or antiquark bound states at finite temperature. In this paper, we are devoted to deriving a rigorous three-dimensional relativistic equation of Dirac-Schrödinger type for quark-antiquark ($q\bar{q}$) bound states at finite temperature. In particular, the interaction kernel in the equation will be given a closed and explicit expression which will be derived by following the procedure described in Refs. [15-17]. This constitutes the main purpose of this paper. Clearly, if the interaction kernel and the equation can be calculated by a suitable nonperturbation method, one can exactly determine at which temperature the quark and antiquark will be deconfined from mesons.

In this paper, we intend to derive the aforementioned equation and kernel from the thermal QCD generating functional formulated in the coherent-state representation. For this derivation, we need first to give a correct expression of the generating functional in the coherent-state representation. This constitutes another purpose of this paper. Here it is necessary to mention that the corresponding path integral expressions for the partition functions and generating functionals given in the previous literature are not correct [18-22]. The incorrectness is due to that in the previous path integral expressions, the integral representing the trace is not separated out on the one hand and the time-dependence of the integrand in the remaining part of the path integral is given incorrectly on the other hand. Such path integral expressions can only be viewed as a formal symbolism because in practical calculations, one has to return to the original discretized forms which lead to the path integral expressions. If one tries to perform an analytical calculation of the path integrals by employing the general methods and formulas of computing functional integrals, one would get a wrong result. The partition functions and generating functionals for many-body systems discussed in quantum statistics were rederived in the coherent-state representation and given correct functional-integral expressions in the author's previous paper [23]. These expressions are consistent with the corresponding coherent-state representations of the transition amplitude and the generating functional in the zero-temperature quantum theory [24-26]. Particularly, when the functional integrals are of Gaussian type, the partition functions and the generating functionals can exactly be calculated by means of the stationary-phase method [25-27]. For the case of interacting systems, the partition functions and finite-temperature Green functions can be conveniently calculated from the generating functionals by the perturbation method. The coherent-state representation of the partition function and the generating functional

given in quantum statistics is now extended to the thermal QCD in this paper, giving a correct formulation for the quantization of the thermal QCD in the coherent-state representation.

The remainder of this paper is arranged as follows. In Sect. 2, we quote the main results given in our previous paper for the quantum statistical mechanics. These results may straightforwardly be extended to the quantum field theory. In Sect. 3, we describe the coherent-state representation of the thermal QCD in the first order (or say, Hamiltonian) formalism. In Sect. 4, the quantization of the thermal QCD is performed in the coherent-state representation by writing out explicitly the generating functional of thermal Green functions. To demonstrate the applicability and correctness of the generating functional, we pay our attention to deriving the perturbative expansion of the generating functional in the coherent-state representation. Section 5 will be used to establish the three-dimensional equation obeyed by the $q\bar{q}$ bound states at finite temperature. Section 6 serves to derive the closed expression of the interaction kernel appearing in the three-dimensional equation. In Sect. 7, the one-gluon exchange kernel and Hamiltonian will be discussed in detail. In the last section, some concluding remarks will be made. In Appendix, the perturbative expansion of the generating functional given in Sect. 4 will be transformed to the corresponding one represented in the position space.

II. PATH INTEGRAL FORMULATION OF QUANTUM STATISTICS IN THE COHERENT-STATE REPRESENTATION

First, we start from the partition function for a grand canonical ensemble which usually is written in the form [19–22]

$$Z = \text{Tr} e^{-\beta \hat{K}} \quad (1)$$

where $\beta = \frac{1}{kT}$ with k and T being the Boltzmann constant and the temperature and

$$\hat{K} = \hat{H} - \mu \hat{N} \quad (2)$$

here μ is the chemical potential, \hat{H} and \hat{N} are the Hamiltonian and particle-number operators respectively. In the coherent-state representation, the trace in Eq. (1) will be represented by an integral over the coherent states. To determine the concrete form of the integral, for simplicity, let us start from an one-dimensional system. Its partition function given in the particle-number representation is

$$Z = \sum_{n=0}^{\infty} \langle n | e^{-\beta \hat{K}} | n \rangle. \quad (3)$$

Then, we use the completeness relation of the coherent states [18–28]

$$\int D(a^* a) | a \rangle \langle a^* | = 1 \quad (4)$$

where $| a \rangle$ denotes a normalized coherent state, i.e., the eigenstate of the annihilation operator \hat{a} with a complex eigenvalue a [18–28]

$$\hat{a} | a \rangle = a | a \rangle \quad (5)$$

whose Hermitian conjugate is

$$\langle a^* | \hat{a}^\dagger = a^* \langle a^* | \quad (6)$$

and $D(a^* a)$ symbolizes the integration measure defined by [18–28]

$$D(a^* a) = \begin{cases} \frac{1}{\pi} da^* da, & \text{for bosons;} \\ da^* da, & \text{for fermions.} \end{cases} \quad (7)$$

In the above, we have used the eigenvalues a and a^* to designate the eigenstates $| a \rangle$ and $\langle a^* |$, respectively. It is emphasized that since we use the normalized eigenfunction of the coherent state whose expression in its own representation will be shown in Eq. (15), the completeness relation in Eq. (4) has the ordinary form as we are familiar with in quantum mechanics. Inserting Eq. (4) into Eq. (3), we have

$$Z = \sum_{n=0}^{\infty} \int D(a^* a) D(a'^* a') \langle n | a' \rangle \langle a'^* | e^{-\beta \hat{K}} | a \rangle \langle a^* | n \rangle \quad (8)$$

where

$$\begin{aligned} \langle a^* | n \rangle &= \frac{1}{\sqrt{n!}} (a^*)^n e^{-a^* a}, \\ \langle n | a' \rangle &= \frac{1}{\sqrt{n!}} (a')^n e^{-a'^* a'} \end{aligned} \quad (9)$$

are the energy eigenfunctions given in the coherent-state representation (Note: for fermions, $n = 0, 1$) [20-26]. The both eigenfunctions commute with the matrix element $\langle a^* | e^{-\beta \hat{K}} | a \rangle$ because the operator $\hat{K}(\hat{a}^+, \hat{a})$ generally is a polynomial of the operator $\hat{a}^+ \hat{a}$ for fermion systems. In view of the expressions in Eq. (9) and the commutation relation [20- 26]

$$a^* a' = \pm a' a^* \quad (10)$$

where the signs "+" and "-" are attributed to bosons and fermions respectively, it is easy to see

$$\langle n | a' \rangle \langle a^* | n \rangle = \langle \pm a^* | n \rangle \langle n | a' \rangle. \quad (11)$$

Substituting Eq. (11) in Eq. (8) and applying the completeness relations for the particle-number states and coherent ones, one may find

$$Z = \int D(a^* a) \langle \pm a^* | e^{-\beta \hat{K}} | a \rangle \quad (12)$$

where the plus and minus signs in front of a^* belong to bosons and fermions respectively.

To evaluate the matrix element in Eq. (12), we may, as usual, divide the "time" interval $[0, \beta]$ into n equal and infinitesimal parts, $\beta = n\varepsilon$. and then insert a completeness relation shown in Eq. (4) at each dividing point. In this way, Eq. (12) may be represented as [19-27]

$$\begin{aligned} Z &= \int D(a^* a) \prod_{i=1}^{n-1} D(a_i^* a_i) \langle \pm a^* | e^{-\varepsilon \hat{K}} | a_{n-1} \rangle \langle a_{n-1}^* | e^{-\varepsilon \hat{K}} | a_{n-2} \rangle \cdots \\ &\quad \times \langle a_{i+1}^* | e^{-\varepsilon \hat{K}} | a_i \rangle \langle a_i^* | e^{-\varepsilon \hat{K}} | a_{i-1} \rangle \cdots \langle a_1^* | e^{-\varepsilon \hat{K}} | a \rangle \end{aligned} \quad (13)$$

Since ε is infinitesimal, we can write

$$e^{-\varepsilon \hat{K}(\hat{a}^+, \hat{a})} \approx 1 - \varepsilon \hat{K}(\hat{a}^+, \hat{a}) \quad (14)$$

where $\hat{K}(\hat{a}^+, \hat{a})$ is assumed to be normally ordered. Noticing this fact, when applying the equations (5) and (6) and the inner product of two coherent states [19-27]

$$\langle a_i^* | a_{i-1} \rangle = e^{-\frac{1}{2} a_i^* a_i - \frac{1}{2} a_{i-1}^* a_{i-1} + a_i^* a_{i-1}} \quad (15)$$

which suits to the both of bosons and fermions, one can get from Eq. (13) that

$$\begin{aligned} Z &= \int D(a^* a) e^{-a^* a} \int \prod_{i=1}^{n-1} D(a_i^* a_i) \exp\{-\varepsilon \sum_{i=1}^n K(a_i^*, a_{i-1}) \\ &\quad + \sum_{i=1}^n a_i^* a_{i-1} - \sum_{i=1}^{n-1} a_i^* a_i\} \end{aligned} \quad (16)$$

where we have set

$$\pm a^* = a_n^*, \quad a = a_0. \quad (17)$$

It is noted that the factor $e^{-a^* a}$ in the first integrand comes from the matrix elements $\langle \pm a^* | a_{n-1} \rangle$ and $\langle a_1^* | a \rangle$ and the last sum in the above exponent is obtained by summing up the common terms $-\frac{1}{2} a_i^* a_i$ and $-\frac{1}{2} a_{i-1}^* a_{i-1}$ appearing in the exponents of the matrix element $\langle a_i^* | a_{i-1} \rangle$ and its adjacent ones $\langle a_{i+1}^* | a_i \rangle$ and $\langle a_{i-1}^* | a_{i-2} \rangle$. As will be seen in Eq. (21), such a summation is essential to give a correct time-dependence of the functional integrand in the partition function. The last two sums in Eq. (16) can be rewritten in the form

$$\begin{aligned}
& \sum_{i=1}^n a_i^* a_{i-1} - \sum_{i=1}^{n-1} a_i^* a_i \\
&= \frac{1}{2} a_n^* a_{n-1} + \frac{1}{2} a_1^* a_0 + \frac{\varepsilon}{2} \sum_{i=1}^{n-1} \left[\left(\frac{a_{i+1}^* - a_i^*}{\varepsilon} \right) a_i - a_i^* \left(\frac{a_i - a_{i-1}}{\varepsilon} \right) \right].
\end{aligned} \tag{18}$$

Upon substituting Eq. (18) in Eq. (16) and taking the limit $\varepsilon \rightarrow 0$, we obtain the path-integral expression of the partition functions as follows:

$$Z = \int D(a^* a) e^{-a^* a} \int \mathfrak{D}(a^* a) e^{I(a^*, a)} \tag{19}$$

where

$$\mathfrak{D}(a^* a) = \begin{cases} \prod_{\tau} \frac{1}{\pi} da^*(\tau) da(\tau), & \text{for bosons;} \\ \prod_{\tau} da^*(\tau) da(\tau), & \text{for fermions} \end{cases} \tag{20}$$

and

$$\begin{aligned}
I(a^*, a) &= \frac{1}{2} a^*(\beta) a(\beta) + \frac{1}{2} a^*(0) a(0) - \int_0^{\beta} d\tau \left[\frac{1}{2} a^*(\tau) \dot{a}(\tau) \right. \\
&\quad \left. - \frac{1}{2} \dot{a}^*(\tau) a(\tau) + K(a^*(\tau), a(\tau)) \right] \\
&= a^*(\beta) a(\beta) - \int_0^{\beta} d\tau [a^*(\tau) \dot{a}(\tau) + K(a^*(\tau), a(\tau))]
\end{aligned} \tag{21}$$

where the last equality is obtained from the first one by a partial integration. In accordance with the definition given in Eq. (17), we see, the path-integral is subject to the following boundary conditions

$$a^*(\beta) = \pm a^*, a(0) = a \tag{22}$$

where the signs "+" and "-" are written respectively for bosons and fermions. Here it is noted that Eq. (22) does not implies $a(\beta) = \pm a$ and $a^*(0) = a^*$. Actually, we have no such boundary conditions.

For the systems with many degrees of freedom, the functional-integral representation of the partition functions may directly be written out from the results given in Eqs. (19) -(22) as long as the eigenvalues a and a^* are understood as column matrices $a = (a_1, a_2, \dots, a_k, \dots)$ and $a^* = (a_1^*, a_2^*, \dots, a_k^*, \dots)$. Written explicitly, we have

$$Z = \int D(a^* a) e^{-a_k^* a_k} \int \mathfrak{D}(a^* a) e^{I(a^*, a)} \tag{23}$$

where

$$D(a^* a) = \begin{cases} \prod_k \frac{1}{\pi} da_k^* da_k & , \text{for bosons;} \\ \prod_k da_k^* da_k & , \text{for fermions,} \end{cases} \tag{24}$$

$$\mathfrak{D}(a^* a) = \begin{cases} \prod_{k\tau} \frac{1}{\pi} da_k^*(\tau) da_k(\tau) & , \text{for bosons;} \\ \prod_{k\tau} da_k^*(\tau) da_k(\tau) & , \text{for fermions} \end{cases} \tag{25}$$

and

$$I(a^*, a) = a_k^*(\beta) a_k(\beta) - \int_0^{\beta} d\tau [a_k^*(\tau) \dot{a}_k(\tau) + K(a_k^*(\tau), a_k(\tau))]. \tag{26}$$

The boundary conditions in Eq. (22) now become

$$a_k^*(\beta) = \pm a_k^*, a_k(0) = a_k. \tag{27}$$

In Eqs. (23) and (26), the repeated indices imply the summations over k . If the k stands for a continuous index as in the case of quantum field theory, the summations will be replaced by integrations over k .

It should be pointed out that in the previous derivation of the coherent-state representation of the partition functions, the authors did not use the expressions given in Eqs. (16) and (18). Instead, the matrix element in Eq. (15) was directly chosen to be the starting point and recast in the form [18-22]

$$\langle a_i^* | a_{i-1} \rangle = \exp\left\{-\frac{\varepsilon}{2}\left[a_i^*\left(\frac{a_i - a_{i-1}}{\varepsilon}\right) - \left(\frac{a_i^* - a_{i-1}^*}{\varepsilon}\right)a_{i-1}\right]\right\}. \quad (28)$$

Substituting the above expression into Eq. (13) and taking the limit $\varepsilon \rightarrow 0$, it follows [18-22]

$$Z = \int \mathfrak{D}(a^* a) \exp\left\{-\int_0^\beta d\tau \left[\frac{1}{2}a^*(\tau)\dot{a}(\tau) - \frac{1}{2}\dot{a}^*(\tau)a(\tau) + K(a^*(\tau), a(\tau))\right]\right\}. \quad (29)$$

Clearly, in the above derivation, the common terms appearing in the exponents of adjacent matrix elements were not combined together. As a result, the time-dependence of the integrand in Eq. (29) could not be given correctly. In comparison with the previous result shown in Eq. (29), the expression written in Eqs. (19)-(21) has two functional integrals. The first integral which represents the trace in Eq. (1) is absent in Eq. (29). The second integral is defined as the same as the integral in Eq. (29); but the integrand are different from each other. In Eq. (19), there occur two additional factors in the integrand: one is $e^{-a^* a}$ which comes from the initial and final states in Eq. (13), another is $e^{\frac{1}{2}[a^*(\beta)a(\beta) + a^*(0)a(0)]}$ in which $a^*(\beta)$ and $a(0)$ are related to the boundary conditions shown in Eq. (22). These additional factors are also absent in Eq. (29). As will be seen soon later, the occurrence of these factors in the functional-integral expression is essential to give correct calculated results.

To demonstrate the correctness of the expression given in Eqs. (23)-(27), let us compute the partition function for the system whose Hamiltonian is of harmonic oscillator-type as we meet in the cases of ideal gases and free fields. In this case,

$$K(a^* a) = \omega_k a_k^* a_k \quad (30)$$

where $\omega_k = \varepsilon_k - \mu$ with ε_k being the particle energy and therefore Eq. (26) becomes

$$I(a^*, a) = a_k^*(\beta)a_k(\beta) - \int_0^\beta d\tau [a_k^*(\tau)\dot{a}_k(\tau) + \omega_k a_k^*(\tau)a_k(\tau)]. \quad (31)$$

By the stationary-phase method which is established based on the property of the Gaussian integral that the integral is equal to the extremum of the integrand which is an exponential function [25-27], we may write

$$\int \mathfrak{D}(a^* a) e^{I(a^*, a)} = e^{I_0(a^*, a)} \quad (32)$$

where $I_0(a^*, a)$ is obtained from $I(a^*, a)$ by replacing the variables $a_k^*(\tau)$ and $a_k(\tau)$ in $I(a^*, a)$ with those values which are determined from the stationary condition $\delta I(a^*, a) = 0$. From this condition and the boundary conditions in Eq. (27) which implies $\delta a_k^*(\beta) = 0$ and $\delta a_k(0) = 0$, it is easy to derive the following equations of motion [24-26]

$$\dot{a}_k(\tau) + \omega_k a_k(\tau) = 0, \quad \dot{a}_k^*(\tau) - \omega_k a_k^*(\tau) = 0. \quad (33)$$

Their solutions satisfying the boundary condition are

$$a_k(\tau) = a_k e^{-\omega_k \tau}, \quad a_k^*(\tau) = \pm a_k^* e^{\omega_k(\tau - \beta)}. \quad (34)$$

On substituting the above solutions into Eq. (31), we obtain

$$I_0(a^*, a) = \pm a_k^* a_k e^{-\omega_k \beta} \quad (35)$$

With the functional integral given in Eqs. (32) and (35), the partition functions in Eq. (23) become

$$Z_0 = \begin{cases} \int D(a^* a) e^{-a_k^* a_k (1 - e^{-\beta \omega_k})} & , \text{ for bosons;} \\ \int D(a^* a) e^{-a_k^* a_k (1 + e^{-\beta \omega_k})} & , \text{ for fermions.} \end{cases} \quad (36)$$

For the boson case, the above integral can directly be calculated by employing the integration formula [18]:

$$\int D(a^* a) e^{-a^*(\lambda a - b)} f(a) = \frac{1}{\lambda} f(\lambda^{-1} b) \quad (37)$$

The result is well-known, as shown in the following [20-22, 29]

$$Z_0 = \prod_k \frac{1}{1 - e^{-\beta\omega_k}} \quad (38)$$

For the fermion case, by using the property of Grassmann algebra and the integration formulas [21-26]:

$$\int da = \int da^* = 0, \quad \int da^* a^* = \int daa = 1 \quad (39)$$

it is easy to compute the integral in Eq. (36) and get the familiar result [21-23,29]

$$Z_0 = \prod_k (1 + e^{-\beta\omega_k}) \quad (40)$$

It is noted that if the stationary-phase method is applied to the functional integral in Eq. (29), one could not get the results as written in Eqs. (38) and (40), showing the incorrectness of the previous functional-integral representation for the partition functions.

Now let us turn to discuss the general case where the Hamiltonian can be split into a free part and an interaction part. Correspondingly, we can write

$$K(a^*, a) = K_0(a^*, a) + H_I(a^*, a) \quad (41)$$

where $K_0(a^*, a)$ is the same as given in Eq. (30) and $H_I(a^*, a)$ is the interaction Hamiltonian. In this case, to evaluate the partition function, it is convenient to define a generating functional through introducing external sources $j_k^*(\tau)$ and $j_k(\tau)$ such that [21-23]

$$\begin{aligned} Z[j^*, j] &= \int D(a^* a) e^{-a^* a} \int \mathfrak{D}(a^* a) \exp\{a_k^*(\beta) a_k(\beta) \\ &\quad - \int_0^\beta d\tau [a_k^*(\tau) \dot{a}_k(\tau) + K(a^* a) - j_k^*(\tau) a_k(\tau) - a_k^*(\tau) j_k(\tau)]\} \\ &= e^{-\int_0^\beta d\tau H_I(\frac{\delta}{\delta j_k^*(\tau)}, \pm \frac{\delta}{\delta j_k(\tau)}} Z_0[j^*, j] \end{aligned} \quad (42)$$

where the sings "+" and "-" in front of $\frac{\delta}{\delta j_k(\tau)}$ refer to bosons and fermions respectively and $Z_0[j^*, j]$ is defined by

$$Z_0[j^*, j] = \int D(a^* a) e^{-a^* a} \int \mathfrak{D}(a^* a) e^{I(a^*, a; j^*, j)} \quad (43)$$

in which

$$\begin{aligned} I(a^*, a; j^*, j) &= a_k^*(\beta) a_k(\beta) - \int_0^\beta d\tau [a_k^*(\tau) \dot{a}_k(\tau) \\ &\quad + \omega_k a_k^*(\tau) a_k(\tau) - j_k^*(\tau) a_k(\tau) - a_k^*(\tau) j_k(\tau)] \end{aligned} \quad (44)$$

Obviously, the integral in Eq. (43) is of Gaussian-type. Therefore, it can be calculated by means of the stationary-phase method as will be shown in detail in Sect. 4.

The exact partition functions can be obtained from the generating functional in Eq. (42) by setting the external sources to be zero

$$Z = Z[j^*, j] |_{j^*=j=0}. \quad (45)$$

In particular, the generating functional is much useful to compute the finite-temperature Green functions. For simplicity, we take the two-point Green function as an example to show this point. In many-body theory, the Green function usually is defined in the operator formalism by [21,29]

$$G_{kl}(\tau_1, \tau_2) = \frac{1}{Z} Tr\{e^{-\beta\hat{K}} T[\hat{a}_k(\tau_1) \hat{a}_l^+(\tau_2)]\} = Tr\{e^{\beta(\Omega - \hat{K})} T[\hat{a}_k(\tau_1) \hat{a}_l^+(\tau_2)]\} \quad (46)$$

where $0 < \tau_1, \tau_2 < \beta$, $\Omega = -\frac{1}{\beta} \ln Z$ is the grand canonical potential, T denotes the "time" ordering operator, $\hat{a}_k(\tau_1)$ and $\hat{a}_l^+(\tau_2)$ represent the annihilation and creation operators respectively. According to the procedure described in Eqs. (12)-(22), it is clear to see that when taking τ_1 and τ_2 at two dividing points and applying the equations (5) and (6), the Green function may be expressed as a functional integral in the coherent-state representation as follows

$$G_{kl}(\tau_1, \tau_2) = \frac{1}{Z} \int D(a^* a) e^{-a^* a} \int \mathfrak{D}(a^* a) a_k(\tau_1) a_l^*(\tau_2) e^{I(a^*, a)}. \quad (47)$$

With the aid of the generating functional defined in Eq. (42), the above Green function may be represented as

$$G_{kl}(\tau_1, \tau_2) = \pm \frac{1}{Z} \frac{\delta^2 Z[j^*, j]}{\delta j_k^*(\tau_1) \delta j_l(\tau_2)} |_{j^*=j=0} \quad (48)$$

where the sings "+" and "-" belong to bosons and fermions respectively.

III. THE COHERENT-STATE REPRESENTATION OF THERMAL QCD HAMILTONIAN AND ACTION

To write out explicitly a path-integral expression of thermal QCD in the coherent-state representation, we first need to formulate the QCD in the coherent-state representation, namely, to give exact expressions of the QCD Hamiltonian and action in the coherent-state representation. For this purpose, we only need to work with the classical fields by using some skilful treatments. Let us start from the effective Lagrangian density of QCD which appears in the path-integral of the zero-temperature QCD [22, 25, 26]

$$\mathcal{L} = \bar{\psi}\{i\gamma^\mu(\partial_\mu - igT^a A_\mu^a) - m\}\psi - \frac{1}{4}F^{a\mu\nu}F_{\mu\nu}^a - \frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2 - \partial^\mu \bar{C}^a D_\mu^{ab} C^b \quad (49)$$

where $T^a = \lambda^a/2$ is the color matrix, ψ and $\bar{\psi}$ represent the quark fields, A_μ^a are the vector potentials of gluon fields, C^a and \bar{C}^a designate the ghost fields,

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c \quad (50)$$

and

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - gf^{abc}A_\mu^c \quad (51)$$

For the sake of simplicity, we work in the Feynman gauge ($\alpha = 1$). It is well-known that in this gauge, the results obtained from the above Lagrangian are equivalent to those derived from the following Lagrangian which is given by applying the Lorentz condition $\partial^\mu A_\mu^a = 0$ to the Lagrangian in Eq. (49),

$$\begin{aligned} \mathcal{L} = & \bar{\psi}\{i\gamma^\mu(\partial_\mu - igT^a A_\mu^a) - m\}\psi - \frac{1}{2}\partial_\mu A_\nu^a \partial^\mu A^{a\nu} - gf^{abc}\partial_\mu A_\nu^a A^{b\mu} A^{c\nu} \\ & - \frac{1}{4}g^2 f^{abc}f^{ade}A^{b\mu} A^{c\nu} A_\mu^d A_\nu^e - \partial^\mu \bar{C}^a \partial_\mu C^b + gf^{abc}\partial^\mu \bar{C}^a C^b A_\mu^c \end{aligned} \quad (52)$$

Here it is noted that the application of the Lorentz condition only changes the form of free part of the gluon Lagrangian, remaining the interaction part of the Lagrangian in Eq. (49) formally unchanged. The above Lagrangian is written in the Minkowski metric where the γ -matrix is defined as $\gamma_0 = \beta$ and $\vec{\gamma} = \beta\vec{\alpha}$ [26]. In the following, it is convenient to represent the Lagrangian in the Euclidean metric with the imaginary time $\tau = it$ where t is the real time.

Since the path-integral in Eq. (42) is established in the first order (or say, Hamiltonian) formalism, to perform the path-integral quantization of thermal QCD in the coherent-state representation, we need to recast the above Lagrangian in the first order form. In doing this, it is necessary to introduce canonical conjugate momentum densities which are defined by [26, 30]

$$\begin{aligned} \Pi_\psi &= \frac{\partial \mathcal{L}}{\partial \partial_t \psi} = i\bar{\psi}\gamma^0 = i\psi^+, \\ \Pi_{\bar{\psi}} &= \frac{\partial \mathcal{L}}{\partial \partial_t \bar{\psi}} = 0, \\ \Pi_\mu^a &= \frac{\partial \mathcal{L}}{\partial \partial_t A_\mu^a} = -\partial_t A_\mu^a + gf^{abc}A_\mu^b A_\mu^c, \\ \Pi^a &= \left(\frac{\partial \mathcal{L}}{\partial \partial_t C^a}\right)_R = -\partial_t \bar{C}^a, \\ \bar{\Pi}^a &= \left(\frac{\partial \mathcal{L}}{\partial \partial_t \bar{C}^a}\right)_L = -\partial_t C^a + gf^{abc}C^b A_\mu^c \end{aligned} \quad (53)$$

where the subscripts R and L mark the right and left-derivatives with respect to the real time respectively. With the above momentum densities, the Lagrangian in Eq. (52) can be represented as

$$\mathcal{L} = \Pi_\psi \partial_t \psi + \Pi^{a\mu} \partial_t A_\mu^a + \Pi^a \partial_t C^a + \partial_t \bar{C}^a \bar{\Pi}^a - \mathcal{H} \quad (54)$$

where

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \quad (55)$$

is the Hamiltonian density in which

$$\mathcal{H}_0 = \bar{\psi}(\vec{\gamma} \cdot \nabla + m)\psi + \frac{1}{2}(\Pi_\mu^a)^2 - \frac{1}{2}A_\mu^a \nabla^2 A_\mu^a - \Pi^a \bar{\Pi}^a + \bar{C}^a \nabla^2 C^a \quad (56)$$

is the free Hamiltonian density and

$$\begin{aligned} \mathcal{H}_I = & ig\bar{\psi}T^a\gamma_\mu A_\mu^a\psi + gf^{abc}(i\Pi_\mu^a A_4^c + \partial_i A_\mu^a A_i^c)A_\mu^b - \frac{1}{4}g^2 f^{abc}f^{ade}A_\mu^b A_\mu^d \\ & \times (A_4^c A_4^e - A_i^c A_i^e) + gf^{abc}(i\Pi^a A_4^c - \partial_i \bar{C}^a A_i^c)C^b \end{aligned} \quad (57)$$

is the interaction Hamiltonian density here the Latin letter i denotes the spatial index. The above Hamiltonian density is written in the Euclidean metric for later convenience. The matrix γ_μ in this metric is defined by $\gamma_4 = \beta$ and $\vec{\gamma} = -i\beta\vec{\alpha}$ [30]. It should be noted that the conjugate quantities Π^a and $\bar{\Pi}^a$ for the ghost fields are respectively defined by the right-derivative and the left one as shown in Eq. (53) because only in this way one can get correct results. This unusual definition originates from the peculiar property of the ghost fields which are scalar fields, but subject to the commutation rule of Grassmann algebra.

In order to derive an expression of the thermal QCD in the coherent-state representation, one should employ the Fourier transformations for the canonical variables of the QCD which are listed below. For the quark field [26, 30],

$$\psi(\vec{x}, \tau) = \int \frac{d^3p}{(2\pi)^{3/2}} [u^s(\vec{p})b_s(\vec{p}, \tau)e^{i\vec{p}\cdot\vec{x}} + v^s(\vec{p})d_s^*(\vec{p}, \tau)e^{-i\vec{p}\cdot\vec{x}}] \quad (58)$$

$$\bar{\psi}(\vec{x}, \tau) = \int \frac{d^3p}{(2\pi)^{3/2}} [\bar{u}^s(\vec{p})b_s^*(\vec{p}, \tau)e^{-i\vec{p}\cdot\vec{x}} + \bar{v}^s(\vec{p})d_s(\vec{p}, \tau)e^{i\vec{p}\cdot\vec{x}}] \quad (59)$$

where $u^s(\vec{p})$ and $v^s(\vec{p})$ are the spinor wave functions satisfying the normalization conditions $u^{s+}(\vec{p})u^s(\vec{p}) = v^{s+}(\vec{p})v^s(\vec{p}) = 1$, $b_s(\vec{p}, \tau)$ and $b_s^*(\vec{p}, \tau)$ are the eigenvalues of the quark annihilation and creation operators $\hat{b}_s(\vec{p}, \tau)$ and $\hat{b}_s^+(\vec{p}, \tau)$ which are defined in the Heisenberg picture, $d_s(\vec{p}, \tau)$ and $d_s^*(\vec{p}, \tau)$ are the corresponding ones for antiquarks. For the gluon field [26, 30],

$$A_\mu^c(\vec{x}, \tau) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\vec{k})}} \varepsilon_\mu^\lambda(\vec{k}) [a_\lambda^c(\vec{k}, \tau)e^{i\vec{k}\cdot\vec{x}} + a_\lambda^{c*}(\vec{k}, \tau)e^{-i\vec{k}\cdot\vec{x}}] \quad (60)$$

where $\varepsilon_\mu^\lambda(\vec{k})$ is the polarization vector and

$$\Pi_\mu^c(\vec{x}, \tau) = i \int \frac{d^3k}{(2\pi)^{3/2}} \sqrt{\frac{\omega(\vec{k})}{2}} \varepsilon_\mu^\lambda(\vec{k}) [a_\lambda^c(\vec{k}, \tau)e^{i\vec{k}\cdot\vec{x}} - a_\lambda^{c*}(\vec{k}, \tau)e^{-i\vec{k}\cdot\vec{x}}] \quad (61)$$

which follows from the definition in Eq. (53) and is consistent with the Fourier representation of free fields. In the above, $a_\lambda^c(\vec{k}, \tau)$ and $a_\lambda^{c*}(\vec{k}, \tau)$ are the eigenvalues of the gluon annihilation and creation operators $\hat{a}_\lambda^c(\vec{k}, \tau)$ and $\hat{a}_\lambda^{c+}(\vec{k}, \tau)$. For the ghost field, we have

$$\bar{C}^a(\vec{x}, \tau) = \int \frac{d^3q}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\vec{q})}} [\bar{c}_a(\vec{q}, \tau)e^{i\vec{q}\cdot\vec{x}} + c_a^*(\vec{q}, \tau)e^{-i\vec{q}\cdot\vec{x}}], \quad (62)$$

$$C^a(\vec{x}, \tau) = \int \frac{d^3q}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\vec{q})}} [c_a(\vec{q}, \tau)e^{i\vec{q}\cdot\vec{x}} + \bar{c}_a^*(\vec{q}, \tau)e^{-i\vec{q}\cdot\vec{x}}], \quad (63)$$

$$\Pi^a(\vec{x}, \tau) = i \int \frac{d^3q}{(2\pi)^{3/2}} \sqrt{\frac{\omega(\vec{q})}{2}} [\bar{c}_a(\vec{q}, \tau)e^{i\vec{q}\cdot\vec{x}} - c_a^*(\vec{q}, \tau)e^{-i\vec{q}\cdot\vec{x}}], \quad (64)$$

and

$$\bar{\Pi}^a(\vec{x}, \tau) = i \int \frac{d^3q}{(2\pi)^{3/2}} \sqrt{\frac{\omega(\vec{q})}{2}} [c_a(\vec{q}, \tau)e^{i\vec{q}\cdot\vec{x}} - \bar{c}_a^*(\vec{q}, \tau)e^{-i\vec{q}\cdot\vec{x}}]. \quad (65)$$

where $c_a(\vec{q}, \tau)$ and $c_a^*(\vec{q}, \tau)$ are the eigenvalues of the ghost particle annihilation and creation operators $\hat{c}_a(\vec{q}, \tau)$ and $\hat{c}_a^+(\vec{q}, \tau)$ and $\bar{c}_a(\vec{q}, \tau)$ and $\bar{c}_a^*(\vec{q}, \tau)$ are the ones for antighost particles.

For simplifying the expressions of the Hamiltonian and action of the thermal QCD, it is convenient to use abbreviation notations. Define

$$b_s^\theta(\vec{p}, \tau) = \begin{cases} b_s(\vec{p}, \tau), & \text{if } \theta = +, \\ d_s^*(\vec{p}, \tau), & \text{if } \theta = -, \end{cases} \quad (66)$$

$$W_s^\theta(\vec{p}) = \begin{cases} (2\pi)^{-3/2} u^s(\vec{p}), & \text{if } \theta = +, \\ (2\pi)^{-3/2} v^s(\vec{p}), & \text{if } \theta = - \end{cases} \quad (67)$$

and furthermore, set $\alpha = (\vec{p}, s, \theta)$ and

$$\sum_\alpha = \sum_{s\theta} \int d^3p, \quad (68)$$

Eqs. (58) and (59) may be represented as

$$\begin{aligned} \psi(\vec{x}, \tau) &= \sum_\alpha W_\alpha b_\alpha(\tau) e^{i\theta \vec{p} \cdot \vec{x}}, \\ \bar{\psi}(\vec{x}, \tau) &= \sum_\alpha \bar{W}_\alpha b_\alpha^*(\tau) e^{-i\theta \vec{p} \cdot \vec{x}}. \end{aligned} \quad (69)$$

Similarly, when we define

$$a_{\lambda\theta}^c(\vec{k}, \tau) = \begin{cases} a_\lambda^c(\vec{k}, \tau), & \text{if } \theta = +, \\ a_\lambda^{c*}(\vec{k}, \tau), & \text{if } \theta = -, \end{cases} \quad (70)$$

$$\begin{aligned} A_{\mu\theta}^{c\lambda}(\vec{k}) &= (2\pi)^{-3/2} (2\omega(\vec{k}))^{-1/2} \epsilon_\mu^\lambda(\vec{k}), \\ \Pi_{\mu\theta}^{c\lambda}(\vec{k}) &= i^\theta (2\pi)^{-3/2} [\omega(\vec{k})/2]^{1/2} \epsilon_\mu^\lambda(\vec{k}) \end{aligned} \quad (71)$$

and furthermore, set $\alpha = (\vec{k}, c, \lambda, \theta)$ and

$$\sum_\alpha = \sum_{c\lambda\theta} \int d^3k, \quad (72)$$

Eqs. (60) and (61) can be written as

$$\begin{aligned} A_\mu^c(\vec{x}, \tau) &= \sum_\alpha A_\mu^\alpha a_\alpha(\tau) e^{i\theta \vec{k} \cdot \vec{x}}, \\ \Pi_\mu^c(\vec{x}, \tau) &= \sum_\alpha \Pi_\mu^\alpha a_\alpha(\tau) e^{i\theta \vec{k} \cdot \vec{x}} \end{aligned} \quad (73)$$

For the ghost fields, if we define

$$c_\alpha^\theta(\vec{q}, \tau) = \begin{cases} \bar{c}_a(\vec{q}, \tau), & \text{if } \theta = +, \\ c_a^*(\vec{q}, \tau), & \text{if } \theta = -, \end{cases} \quad (74)$$

$$\begin{aligned} G_\theta(\vec{q}) &= (2\pi)^{-3/2} [2\omega(\vec{q})]^{-1/2}, \\ \Pi_\theta(\vec{q}) &= i^\theta (2\pi)^{-3/2} [\omega(\vec{q})/2]^{1/2}, \end{aligned} \quad (75)$$

and furthermore set $\alpha = (\vec{q}, a, \theta)$ and

$$\sum_\alpha = \sum_{a\theta} \int d^3q \quad (76)$$

then, Eqs. (62)-(65) will be expressed as

$$\begin{aligned} \bar{C}^a(\vec{x}, \tau) &= \sum_\alpha G_\alpha c_\alpha(\tau) e^{i\theta \vec{q} \cdot \vec{x}} \\ C^a(\vec{x}, \tau) &= \sum_\alpha G_\alpha c_\alpha^*(\tau) e^{-i\theta \vec{q} \cdot \vec{x}} \\ \Pi^a(\vec{x}, \tau) &= \sum_\alpha \Pi_\alpha c_\alpha(\tau) e^{i\theta \vec{q} \cdot \vec{x}} \\ \bar{\Pi}^a(\vec{x}, \tau) &= \sum_\alpha \Pi_\alpha c_\alpha^*(\tau) e^{-i\theta \vec{q} \cdot \vec{x}} \end{aligned} \quad (77)$$

Upon substituting Eqs. (69), (73) and (77) into Eqs. (56) and (57), it is not difficult to get

$$H_0(\tau) = \int d^3x \mathcal{H}_0(x) = \sum_{\alpha} \theta_{\alpha} \varepsilon_{\alpha} b_{\alpha}^*(\tau) b_{\alpha}(\tau) + \frac{1}{2} \sum_{\alpha} \omega_{\alpha} a_{\alpha}^*(\tau) a_{\alpha}(\tau) + \sum_{\alpha} \omega_{\alpha} c_{\alpha}^*(\tau) c_{\alpha}(\tau) \quad (78)$$

and

$$H_I(\tau) = \int d^3x \mathcal{H}_I(x) = \sum_{\alpha\beta\gamma} A(\alpha\beta\gamma) b_{\alpha}^*(\tau) b_{\beta}(\tau) a_{\gamma}(\tau) + \sum_{\alpha\beta\gamma} B(\alpha\beta\gamma) a_{\alpha}(\tau) a_{\beta}(\tau) a_{\gamma}(\tau) + \sum_{\alpha\beta\gamma\delta} C(\alpha\beta\gamma\delta) a_{\alpha}(\tau) a_{\beta}(\tau) a_{\gamma}(\tau) a_{\delta}(\tau) + \sum_{\alpha\beta\gamma} D(\alpha\beta\gamma) c_{\alpha}^*(\tau) c_{\beta}(\tau) a_{\gamma}(\tau) \quad (79)$$

which are the QCD Hamiltonian given in the coherent state representation. In Eq. (78), the first, second and third terms are the free Hamiltonians for quarks, gluons and ghost particles respectively where $\theta_{\alpha} \equiv \theta$, $\varepsilon_{\alpha} = (\vec{p}^2 + m^2)^{1/2}$ is the quark energy, $\omega_{\alpha} = |\vec{k}|$ is the energy for a gluon or a ghost particle. In Eq. (79), the first term is the interaction Hamiltonian between quarks and gluons, the second and third terms are the interaction Hamiltonian among gluons and the fourth term represents the interaction Hamiltonian between ghost particles and gluons. The coefficient functions in Eq. (79) are defined as follows:

$$A(\alpha\beta\gamma) = ig(2\pi)^3 \delta^3(\theta_{\alpha} \vec{p}_{\alpha} - \theta_{\beta} \vec{p}_{\beta} - \theta_{\gamma} \vec{k}_{\gamma}) \bar{W}_{s_{\alpha}}^{\theta_{\alpha}}(\vec{p}_{\alpha}) T^a \gamma_{\mu} W_{s_{\alpha}}^{\theta_{\beta}}(\vec{p}_{\beta}) A_{\mu\theta_{\gamma}}^{a\lambda_{\gamma}}(\vec{k}_{\gamma}), \quad (80)$$

$$B(\alpha\beta\gamma) = ig(2\pi)^3 \delta^3(\theta_{\alpha} \vec{k}_{\alpha} + \theta_{\beta} \vec{k}_{\beta} + \theta_{\gamma} \vec{k}_{\gamma}) f^{abc} [\Pi_{\mu\theta_{\alpha}}^{a\lambda_{\alpha}}(\vec{k}_{\alpha}) \times A_{4\theta_{\gamma}}^{c\lambda_{\gamma}}(\vec{k}_{\gamma}) + \theta_{\alpha} k_i^{\alpha} A_{\mu\theta_{\alpha}}^{a\lambda_{\alpha}}(\vec{k}_{\alpha}) A_{i\theta_{\gamma}}^{c\lambda_{\gamma}}(\vec{k}_{\gamma})] A_{\mu\theta_{\beta}}^{b\lambda_{\beta}}(\vec{k}_{\beta}), \quad (81)$$

$$C(\alpha\beta\gamma\delta) = -\frac{1}{4} g^2 (2\pi)^3 \delta^3(\theta_{\alpha} \vec{k}_{\alpha} + \theta_{\beta} \vec{k}_{\beta} + \theta_{\rho} \vec{k}_{\rho} + \theta_{\sigma} \vec{k}_{\sigma}) f^{abc} f^{ade} \times A_{\mu\theta_{\alpha}}^{b\lambda_{\alpha}}(\vec{k}_{\alpha}) A_{\mu\theta_{\beta}}^{d\lambda_{\beta}}(\vec{k}_{\beta}) [A_{4\theta_{\rho}}^{c\lambda_{\rho}}(\vec{k}_{\rho}) A_{4\theta_{\sigma}}^{e\lambda_{\sigma}}(\vec{k}_{\sigma}) - A_{i\theta_{\rho}}^{c\lambda_{\rho}}(\vec{k}_{\rho}) A_{i\theta_{\sigma}}^{e\lambda_{\sigma}}(\vec{k}_{\sigma})] \quad (82)$$

and

$$D(\alpha\beta\gamma) = ig(2\pi)^3 \delta^3(\theta_{\alpha} \vec{q}_{\alpha} - \theta_{\beta} \vec{q}_{\beta} - \theta_{\gamma} \vec{k}_{\gamma}) f^{abc} G_{\theta_{\alpha}}^a(\vec{q}_{\alpha}) \times [\Pi_{\theta_{\beta}}^b(\vec{q}_{\beta}) A_{4\theta_{\gamma}}^{c\lambda_{\gamma}}(\vec{k}_{\gamma}) - \theta_{\alpha} k_i^{\alpha} G_{\theta_{\beta}}^b(\vec{q}_{\beta}) A_{i\theta_{\gamma}}^{c\lambda_{\gamma}}(\vec{k}_{\gamma})]. \quad (83)$$

It is emphasized that the expressions in Eqs. (78) and (79) are just the Hamiltonian of QCD appearing in the path-integral as shown in Eq. (42) where all the creation and annihilation operators in the Hamiltonian (which are written in a normal product) are replaced by their eigenvalues.

To write the path-integral of thermal QCD, we need also an expression of action S given in the coherent state representation. This action can be obtained by using the Lagrangian density shown in Eq. (54). By partial integration and considering the following boundary conditions of the fields [20-22]:

$$\psi(\vec{x}, 0) = \psi(\vec{x}), \quad \bar{\psi}(\vec{x}, 0) = \bar{\psi}(\vec{x}), \quad \psi(\vec{x}, \beta) = -\psi(\vec{x}), \quad \bar{\psi}(\vec{x}, \beta) = -\bar{\psi}(\vec{x}), \quad (84)$$

$$A_{\mu}^c(\vec{x}, 0) = A_{\mu}^c(\vec{x}, \beta) = A_{\mu}^c(\vec{x}), \quad \Pi_{\mu}^c(\vec{x}, 0) = \Pi_{\mu}^c(\vec{x}, \beta) = \Pi_{\mu}^c(\vec{x}) \quad (85)$$

and

$$\bar{C}^a(\vec{x}, 0) = \bar{C}^a(\vec{x}, \beta) = \bar{C}^a(\vec{x}), \quad C^a(\vec{x}, 0) = C^a(\vec{x}, \beta) = C^a(\vec{x}), \quad \bar{\Pi}^a(\vec{x}, 0) = \bar{\Pi}^a(\vec{x}, \beta) = \bar{\Pi}^a(\vec{x}), \quad \Pi^a(\vec{x}, 0) = \Pi^a(\vec{x}, \beta) = \Pi^a(\vec{x}), \quad (86)$$

the action given by the Lagrangian density in Eq. (54) can be represented in the form

$$S = \int_0^{\beta} d\tau \int d^3x \{ \frac{1}{2} [\psi^+(\vec{x}, \tau) \dot{\psi}(\vec{x}, \tau) - \dot{\psi}^+(\vec{x}, \tau) \psi(\vec{x}, \tau)] + \frac{i}{2} [\Pi_{\mu}^c(\vec{x}, \tau) \dot{A}_{\mu}^c(\vec{x}, \tau) - \dot{\Pi}_{\mu}^c(\vec{x}, \tau) A_{\mu}^c(\vec{x}, \tau)] + \frac{i}{2} [\Pi_a(\vec{x}, \tau) \dot{C}_a(\vec{x}, \tau) - \dot{\Pi}_a(\vec{x}, \tau) C_a(\vec{x}, \tau)] + \bar{C}_a(\vec{x}, \tau) \dot{\Pi}_a(\vec{x}, \tau) - \bar{\Pi}_a(\vec{x}, \tau) \dot{C}_a(\vec{x}, \tau) \} - \mathcal{H}(\vec{x}, \tau) \} \quad (87)$$

where the first relation in Eq. (53) has been used and the symbol " \cdot " in $\dot{\psi}(\vec{x}, \tau), \dot{A}_\mu^c(\vec{x}, \tau) \dots$ now denotes the derivatives of the fields with respect to the imaginary time τ . It is stressed here that only the above expression is appropriate to use for deriving the coherent-state representation of the action by making use of the Fourier expansions written in Eqs. (58)-(65). On inserting Eqs. (58)-(65) into Eq. (87), it is not difficult to get

$$\begin{aligned} S = & - \int_0^\beta d\tau \{ \int d^3k \{ \frac{1}{2} [b_s^*(\vec{k}, \tau) \dot{b}_s(\vec{k}, \tau) - \dot{b}_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau)] + \frac{1}{2} [d_s^*(\vec{k}, \tau) \dot{d}_s(\vec{k}, \tau) \\ & - \dot{d}_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau)] + \frac{1}{2} [a_\lambda^{c*}(\vec{k}, \tau) \dot{a}_\lambda^c(\vec{k}, \tau) - \dot{a}_\lambda^{c*}(\vec{k}, \tau) a_\lambda^c(\vec{k}, \tau)] + \frac{1}{2} [\bar{c}_a^*(\vec{k}, \tau) \dot{\bar{c}}_a(\vec{k}, \tau) \\ & - \dot{\bar{c}}_a^*(\vec{k}, \tau) \bar{c}_a(\vec{k}, \tau) - c_a^*(\vec{k}, \tau) \dot{c}_a(\vec{k}, \tau) + \dot{c}_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau)] \} + H(\tau) \} \\ = & -S_E \end{aligned} \quad (88)$$

where $H(\tau)$ is given by the sum of the Hamiltonians in Eqs. (78) and (79) and S_E is the action defined in the Euclidean metric. It is noted that if one considers a grand canonical ensemble of QCD, the Hamiltonian in Eq. (88) should be replaced by $K(\tau)$ defined in Eq. (2). Employing the abbreviation notation as denoted in Eqs. (66), (70) and (74) and letting q_α stand for $(a_\alpha, b_\alpha, c_\alpha)$, the action may be compactly represented as

$$S_E = \int_0^\beta d\tau \{ \sum_\alpha \frac{1}{2} [q_\alpha^*(\tau) \circ \dot{q}_\alpha(\tau) - \dot{q}_\alpha^*(\tau) \circ q_\alpha(\tau)] + H(\tau) \} \quad (89)$$

where we have defined

$$q_\alpha^* \circ q_\alpha = a_{\alpha-} a_{\alpha+} + b_\alpha^* b_\alpha + \theta_\alpha c_\alpha^* c_\alpha \quad (90)$$

It is emphasized that the $\theta_\alpha = \pm$ is now contained in the subscript α . Therefore, each α may take α^+ and/or α^- as the first term in Eq. (90) does.

IV. GENERATING FUNCTIONAL OF GREEN FUNCTIONS FOR THERMAL QCD

With the action S_E given in the preceding section, the quantization of the thermal QCD in the coherent-state representation is easily implemented by writing out its generating functional of thermal Green functions. According to the general formula shown in Eq. (42), the QCD generating functional can be formulated as

$$\begin{aligned} Z[j] = & \int D(q^* q) e^{-q^* \cdot q} \int \mathcal{D}(q^* q) \exp \{ \frac{1}{2} [q^*(\beta) \cdot q(\beta) \\ & - q^*(0) \cdot q(0)] - S_E + \int_0^\beta d\tau j^*(\tau) \cdot q(\tau) \} \end{aligned} \quad (91)$$

where we have defined

$$q^* \cdot q = \frac{1}{2} a_\alpha^* a_\alpha + \theta_\alpha b_\alpha^* b_\alpha + c_\alpha^* c_\alpha \quad (92)$$

and

$$j^* \cdot q = \xi_\alpha^* a_\alpha + \theta_\alpha (\eta_\alpha^* b_\alpha + b_\alpha^* \eta_\alpha + \zeta_\alpha^* c_\alpha + c_\alpha^* \zeta_\alpha) \quad (93)$$

here ξ_α, η_α and ζ_α are the sources for gluons, quarks and ghost particles respectively and the repeated index implies summation. It is noted that the product $q^* \cdot q$ defined above is different from the $q_\alpha^* \circ q_\alpha$ defined in Eq. (90) in the terms for quarks and ghost particles and the subscript α in Eqs. (92) and (93) is also defined by containing $\theta_\alpha = \pm$. In what follows, we assign α^\pm to represent the α with $\theta_\alpha = \pm$. According to this notation, the sources in Eq. (93) are specifically defined as follows:

$$\begin{aligned} \xi_{\alpha+} &= \xi_\alpha, \quad \xi_{\alpha-} = \xi_\alpha^* \\ \eta_{\alpha+} &= \eta_\alpha, \quad \eta_{\alpha-} = \bar{\eta}_\alpha^* \\ \zeta_{\alpha+} &= \zeta_\alpha, \quad \zeta_{\alpha-} = \bar{\zeta}_\alpha^* \end{aligned} \quad (94)$$

where the subscript α on the right hand side of each equality no longer contains θ_α and the gluon term in Eq. (92) $(1/2)a_\alpha^* a_\alpha$ may be replaced by $a_{\alpha-} a_{\alpha+}$. The integration measures $D(q^* q)$ and $\mathcal{D}(q^* q)$ are defined as in Eqs. (24) and (25).

The generating functional in Eq. (91) is nonperturbative. Now we are interested in describing the perturbation method of calculating the QCD generating functional. Since the Hamiltonian can be split into two parts $H_0(\tau)$ and

$H_I(\tau)$ as shown in Eqs. (78) and (79), the generating functional in Eq. (91) may be perturbatively represented in the form

$$Z[j] = \exp\left\{-\int_0^\beta d\tau H_I\left(\frac{\delta}{\delta j(\tau)}\right)\right\} Z^0[j] \quad (95)$$

where $Z^0[j]$ is the generating functional for the free system and the exponential may be expanded in a Taylor series. In the above, the commutativity between H_I and $Z^0[j]$ has been considered. Obviously, the $Z^0[j]$ can be written as

$$Z^0[j] = Z_g^0[\xi] Z_q^0[\eta] Z_c^0[\zeta] \quad (96)$$

where $Z_g^0[\xi]$, $Z_q^0[\eta]$ and $Z_c^0[\zeta]$ are the generating functionals contributed from the free Hamiltonians of gluons, quarks and ghost particles respectively. They are separately and specifically described below.

In view of the expressions in Eqs. (91), (88) and (78), the generating functional $Z_g^0[\xi]$ is of the form

$$Z_g^0[\xi] = \int D(a^* a) \exp\left\{-\int d^3 k a_\lambda^*(\vec{k}) a_\lambda(\vec{k})\right\} \times \int \mathfrak{D}(a^* a) \exp\{I_g(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda)\} \quad (97)$$

where

$$I_g(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda) = \int d^3 k \frac{1}{2} [a_\lambda^*(\vec{k}, \beta) a_\lambda(\vec{k}, \beta) + a_\lambda^*(\vec{k}, 0) a_\lambda(\vec{k}, 0)] - \int_0^\beta d\tau \int d^3 k \left\{ \frac{1}{2} [a_\lambda^*(\vec{k}, \tau) \dot{a}_\lambda(\vec{k}, \tau) - \dot{a}_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k}, \tau)] + \omega(\vec{k}) a_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k}, \tau) - \xi_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k}, \tau) - a_\lambda^*(\vec{k}, \tau) \xi_\lambda(\vec{k}, \tau) \right\} \quad (98)$$

and

$$D(a^* a) = \prod_{\vec{k}\lambda} \frac{1}{\pi} da_\lambda^*(\vec{k}) da_\lambda(\vec{k}), \quad \mathfrak{D}(a^* a) = \prod_{\vec{k}\lambda\tau} \frac{1}{\pi} da_\lambda^*(\vec{k}, \tau) da_\lambda(\vec{k}, \tau). \quad (99)$$

The subscript λ in the above is now assigned to denote polarization and color. When we perform a partial integration, Eq. (98) becomes

$$I_g(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda) = \int d^3 k a_\lambda^*(\vec{k}, \beta) a_\lambda(\vec{k}, \beta) - \int_0^\beta d\tau \int d^3 k \{ a_\lambda^*(\vec{k}, \tau) \dot{a}_\lambda(\vec{k}, \tau) + \omega(\vec{k}) a_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k}, \tau) - \xi_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k}, \tau) - a_\lambda^*(\vec{k}, \tau) \xi_\lambda(\vec{k}, \tau) \}. \quad (100)$$

For the generating functional $Z_q^0[\eta]$, we can write

$$Z_q^0[\eta] = \int D(b^* b d^* d) \exp\left\{-\int d^3 k [b_s^*(\vec{k}) b_s(\vec{k}) + d_s^*(\vec{k}) d_s(\vec{k})]\right\} \times \int \mathfrak{D}(b^* b d^* d) \exp\{I_q(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s)\} \quad (101)$$

where

$$I_q(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s) = \int d^3 k \frac{1}{2} [b_s^*(\vec{k}, \beta) b_s(\vec{k}, \beta) + d_s^*(\vec{k}, \beta) d_s(\vec{k}, \beta) + b_s^*(\vec{k}, 0) b_s(\vec{k}, 0) + d_s^*(\vec{k}, 0) d_s(\vec{k}, 0)] - \int_0^\beta d\tau \int d^3 k \left\{ \frac{1}{2} [b_s^*(\vec{k}, \tau) \dot{b}_s(\vec{k}, \tau) - \dot{b}_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau)] + \frac{1}{2} [d_s^*(\vec{k}, \tau) \dot{d}_s(\vec{k}, \tau) - \dot{d}_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau)] + \varepsilon(\vec{k}) [b_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau) + d_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau)] - [\eta_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau) + b_s^*(\vec{k}, \tau) \eta_s(\vec{k}, \tau) + \bar{\eta}_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau) + d_s^*(\vec{k}, \tau) \bar{\eta}_s(\vec{k}, \tau)] \right\} \quad (102)$$

and

$$D(b^* b d^* d) = \prod_{\vec{k}s} db_s^*(\vec{k}) db_s(\vec{k}) dd_s^*(\vec{k}) dd_s(\vec{k}), \quad \mathfrak{D}(b^* b d^* d) = \prod_{\vec{k}s\tau} db_s^*(\vec{k}, \tau) db_s(\vec{k}, \tau) dd_s^*(\vec{k}, \tau) dd_s(\vec{k}, \tau) \quad (103)$$

in which the subscript s stands for spin, color and flavor. By a partial integration over τ , Eq. (102) may be given a simpler expression

$$\begin{aligned}
I_q(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s) = & \int d^3 k [b_s^*(\vec{k}, \beta) b_s(\vec{k}, \beta) + d_s^*(\vec{k}, \beta) d_s(\vec{k}, \beta)] \\
& - \int_0^\beta d\tau \int d^3 k \{ b_s^*(\vec{k}, \tau) \dot{b}_s(\vec{k}, \tau) + d_s^*(\vec{k}, \tau) \dot{d}_s(\vec{k}, \tau) \\
& + \varepsilon(\vec{k}) [b_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau) + d_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau)] - [\eta_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau) \\
& + b_s^*(\vec{k}, \tau) \eta_s(\vec{k}, \tau) + \bar{\eta}_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau) + d_s^*(\vec{k}, \tau) \bar{\eta}_s(\vec{k}, \tau)] \}.
\end{aligned} \tag{104}$$

As for the generating functional $Z_c^0[\zeta]$, we have

$$\begin{aligned}
Z_c^0[\zeta] = & \int D(\bar{c}^* \bar{c} c c^*) \exp\{-\int d^3 k [\bar{c}_a^*(\vec{k}) \bar{c}_a(\vec{k}) - c_a^*(\vec{k}) c_a(\vec{k})]\} \\
& \times \int \mathfrak{D}(\bar{c}^* \bar{c} c c^*) \exp\{I_c(c_a^*, c_a, \bar{c}_a^*, \bar{c}_a; \zeta_a^*, \zeta_a, \bar{\zeta}_a^*, \bar{\zeta}_a)\}
\end{aligned} \tag{105}$$

where

$$\begin{aligned}
I_c(c_a^*, c_a, \bar{c}_a^*, \bar{c}_a; \zeta_a^*, \zeta_a, \bar{\zeta}_a^*, \bar{\zeta}_a) = & \int d^3 k \frac{1}{2} [\bar{c}_a^*(\vec{k}, \beta) \bar{c}_a(\vec{k}, \beta) - c_a^*(\vec{k}, \beta) c_a(\vec{k}, \beta) \\
& + \bar{c}_a^*(\vec{k}, 0) \bar{c}_a(\vec{k}, 0) - c_a^*(\vec{k}, 0) c_a(\vec{k}, 0)] - \int_0^\beta d\tau \int d^3 k \{ \frac{1}{2} [\bar{c}_a^*(\vec{k}, \tau) \dot{\bar{c}}_a(\vec{k}, \tau) \\
& - \dot{\bar{c}}_a^*(\vec{k}, \tau) \bar{c}_a(\vec{k}, \tau)] - \frac{1}{2} [c_a^*(\vec{k}, \tau) \dot{c}_a(\vec{k}, \tau) - \dot{c}_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau)] \\
& + \omega(\vec{k}) [\bar{c}_a^*(\vec{k}, \tau) \bar{c}_a(\vec{k}, \tau) - c_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau)] - [\zeta_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau) \\
& + c_a^*(\vec{k}, \tau) \zeta_a(\vec{k}, \tau) + \bar{\zeta}_a^*(\vec{k}, \tau) \bar{c}_a(\vec{k}, \tau) + \bar{c}_a^*(\vec{k}, \tau) \bar{\zeta}_a(\vec{k}, \tau)] \}
\end{aligned} \tag{106}$$

and

$$\begin{aligned}
D(\bar{c}^* \bar{c} c c^*) = & \prod_{\vec{k}a} d\bar{c}_a^*(\vec{k}) d\bar{c}_a(\vec{k}) dc_a(\vec{k}) dc_a^*(\vec{k}), \\
\mathfrak{D}(\bar{c}^* \bar{c} c c^*) = & \prod_{\vec{k}a\tau} d\bar{c}_a^*(\vec{k}, \tau) d\bar{c}_a(\vec{k}, \tau) dc_a(\vec{k}, \tau) dc_a^*(\vec{k}, \tau)
\end{aligned} \tag{107}$$

in which the subscript a is a color index. After a partial integration, Eq. (106) is reduced to

$$\begin{aligned}
I_c(c_a^*, c_a, \bar{c}_a^*, \bar{c}_a; \zeta_a^*, \zeta_a, \bar{\zeta}_a^*, \bar{\zeta}_a) = & \int d^3 k [\bar{c}_a^*(\vec{k}, \beta) \bar{c}_a(\vec{k}, \beta) - c_a^*(\vec{k}, \beta) c_a(\vec{k}, \beta)] \\
& - \int_0^\beta d\tau \int d^3 k \{ \bar{c}_a^*(\vec{k}, \tau) \dot{\bar{c}}_a(\vec{k}, \tau) - c_a^*(\vec{k}, \tau) \dot{c}_a(\vec{k}, \tau) + \omega(\vec{k}) [\bar{c}_a^*(\vec{k}, \tau) \bar{c}_a(\vec{k}, \tau) \\
& - c_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau)] - [\zeta_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau) + c_a^*(\vec{k}, \tau) \zeta_a(\vec{k}, \tau) + \bar{\zeta}_a^*(\vec{k}, \tau) \bar{c}_a(\vec{k}, \tau) \\
& + \bar{c}_a^*(\vec{k}, \tau) \bar{\zeta}_a(\vec{k}, \tau)] \}.
\end{aligned} \tag{108}$$

Here it is noted that all the terms related to the quantities c_a^* and c_a are opposite in sign to the terms related to the \bar{c}_a^* and \bar{c}_a and, correspondingly, the definitions of the integration measures for these quantities, as shown in Eq. (107), are different from each other in the order of the differentials.

The generating functionals in Eqs. (97), (101) and (105) are all of Gaussian-type, therefore, they can exactly be calculated by the stationary-phase method. First, we calculate the functional integral $Z_g^0[\xi]$. According to the stationary-phase method, the functional $Z_g^0[\xi]$ can be represented in the form

$$Z_g^0[\xi] = \int D(a^* a) \exp\{-\int d^3 k [a_\lambda^*(\vec{k}) a_\lambda(\vec{k}) + I_g^0(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda)]\} \tag{109}$$

where $I_g^0(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda)$ is given by the stationary condition $\delta I_g(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda) = 0$. By this condition and the boundary condition [20-22]:

$$a_\lambda^*(\vec{k}, \beta) = a_\lambda^*(\vec{k}), \quad a_\lambda(\vec{k}, 0) = a_\lambda(\vec{k}), \tag{110}$$

one may derive from Eq. (98) or (100) the following inhomogeneous equations of motion [23-26]:

$$\begin{aligned}
\dot{a}_\lambda(\vec{k}, \tau) + \omega(\vec{k}) a_\lambda(\vec{k}, \tau) &= \xi_\lambda(\vec{k}, \tau), \\
\dot{a}_\lambda^*(\vec{k}, \tau) - \omega(\vec{k}) a_\lambda^*(\vec{k}, \tau) &= -\xi_\lambda^*(\vec{k}, \tau).
\end{aligned} \tag{111}$$

In accordance with the general method of solving such a kind of equations, one may first solve the homogeneous linear equations as written in Eq. (33). Based on the solutions shown in Eq. (34) and the boundary condition denoted in Eq. (110), one may assume [23-26]

$$\begin{aligned}
a_\lambda(\vec{k}, \tau) &= [a_\lambda(\vec{k}) + u_\lambda(\vec{k}, \tau)] e^{-\omega(\vec{k})\tau}, \\
a_\lambda^*(\vec{k}, \tau) &= [a_\lambda^*(\vec{k}) + u_\lambda^*(\vec{k}, \tau)] e^{\omega(\vec{k})(\tau-\beta)}
\end{aligned} \tag{112}$$

where the unknown functions $u_\lambda(\vec{k}, \tau)$ and $u_\lambda^*(\vec{k}, \tau)$ are required to satisfy the boundary conditions [23-26]:

$$u_\lambda(\vec{k}, 0) = u_\lambda(\vec{k}, \beta) = u_\lambda^*(\vec{k}, 0) = u_\lambda^*(\vec{k}, \beta) = 0. \quad (113)$$

Inserting Eq. (112) into Eq. (111), we find

$$\begin{aligned} \dot{u}_\lambda(\tau) &= \xi_\lambda(\vec{k}, \tau) e^{\omega(\vec{k})\tau}, \\ \dot{u}_\lambda^*(\tau) &= -\xi_\lambda^*(\vec{k}, \tau) e^{\omega(\vec{k})(\beta-\tau)}. \end{aligned} \quad (114)$$

Integrating these two equations and applying the boundary conditions in Eq. (113), one can get

$$\begin{aligned} u_\lambda(\vec{k}, \tau) &= \int_0^\tau d\tau' e^{\omega(\vec{k})\tau'} \xi_\lambda(\vec{k}, \tau'), \\ u_\lambda^*(\vec{k}, \tau) &= -\int_\tau^\beta d\tau' e^{\omega(\vec{k})(\beta-\tau')} \xi_\lambda^*(\vec{k}, \tau'). \end{aligned} \quad (115)$$

Substitution of these solutions in Eq. (112) yields [23-26]

$$\begin{aligned} a_\lambda(\vec{k}, \tau) &= a_\lambda(\vec{k}) e^{-\omega(\vec{k})\tau} + \int_0^\tau d\tau' e^{-\omega(\vec{k})(\tau-\tau')} \xi_\lambda(\vec{k}, \tau'), \\ a_\lambda^*(\vec{k}, \tau) &= a_\lambda^*(\vec{k}) e^{\omega(\vec{k})(\tau-\beta)} + \int_\tau^\beta d\tau' e^{\omega(\vec{k})(\tau-\tau')} \xi_\lambda^*(\vec{k}, \tau'). \end{aligned} \quad (116)$$

When Eq. (116) is inserted into Eq. (98) or Eq. (100), one may obtain the $I_g^0(a_\lambda^*, a_\lambda; \xi_\lambda^*, \xi_\lambda)$ which leads to another expression of Eq. (109) like this

$$\begin{aligned} Z_g^0[\xi] &= \int D(a^* a) \exp\{-\int d^3k [a_\lambda^*(\vec{k}) a_\lambda(\vec{k}) (1 - e^{-\beta\omega(\vec{k})}) - a_\lambda^*(\vec{k}) e^{-\beta\omega(\vec{k})} \\ &\quad \times \int_0^\beta d\tau e^{\omega(\vec{k})\tau} \xi_\lambda(\vec{k}, \tau) - \int_0^\beta d\tau e^{-\omega(\vec{k})\tau} \xi_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k})] \\ &\quad + \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k \xi_\lambda^*(\vec{k}, \tau_1) \theta(\tau_1 - \tau_2) e^{-\omega(\vec{k})(\tau_1 - \tau_2)} \xi_\lambda(\vec{k}, \tau_2)\}. \end{aligned} \quad (117)$$

When we set

$$\begin{aligned} \lambda &= 1 - e^{-\beta\omega(\vec{k})}, \\ b &= e^{-\beta\omega(\vec{k})} \int_0^\beta d\tau e^{\omega(\vec{k})\tau} \xi_\lambda(\vec{k}, \tau), \\ f(a) &= \int_0^\beta d\tau e^{-\omega(\vec{k})\tau} \xi_\lambda^*(\vec{k}, \tau) a_\lambda(\vec{k}), \end{aligned} \quad (118)$$

by employing the formula denoted in Eq. (37), the integral over $a_\lambda^*(\vec{k})$ and $a_\lambda(\vec{k})$ in Eq. (117) can easily be calculated. The result is

$$Z_g^0[\xi] = Z_g^0 \exp\{-\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k \xi_a^{\lambda*}(\vec{k}, \tau_1) \Delta_{\lambda\lambda'}^{aa'}(\vec{k}, \tau_1 - \tau_2) \xi_{a'}^{\lambda'}(\vec{k}, \tau_2)\} \quad (119)$$

where

$$Z_g^0 = \prod_{\vec{k} \lambda a} [1 - e^{-\beta\omega(\vec{k})}]^{-1} = \prod_{\vec{k} a} [1 - e^{-\beta\omega(\vec{k})}]^{-4} \quad (120)$$

is precisely the partition function contributed from the free gluons [21-24, 29] and

$$\Delta_{\lambda\lambda'}^{aa'}(\vec{k}, \tau_1 - \tau_2) = g_{\lambda\lambda'} \delta^{aa'} \Delta_g(\vec{k}, \tau_1 - \tau_2) \quad (121)$$

with

$$\Delta_g(\vec{k}, \tau_1 - \tau_2) = \theta(\tau_1 - \tau_2) - (1 - e^{\beta\omega(\vec{k})})^{-1} e^{-\omega(\vec{k})(\tau_1 - \tau_2)} \quad (122)$$

is the free gluon propagator given in the Feynman gauge and in the Minkowski metric {Note: in Euclidean metric, $g_{\lambda\lambda'} \rightarrow -\delta_{\lambda\lambda'}$ }. In Eq. (119), the color index "a" has been explicitly written out and the λ now merely designates the polarization index. In the other expressions, we still use λ to mark the both of color and polarization indices for simplicity. When we interchange the integration variables τ_1 and τ_2 and make a transformation $\vec{k} \rightarrow -\vec{k}$ in Eq. (119), by considering the relation

$$\xi_\lambda^*(\vec{k}, \tau) = \xi_\lambda(-\vec{k}, \tau) \quad (123)$$

which will be interpreted in the Appendix, one may find that the propagator in Eq. (122) can be represented in the form

$$\Delta_g(\vec{k}, \tau_1 - \tau_2) = \frac{1}{2} [\bar{n}_b(\vec{k}) e^{-\omega(\vec{k})|\tau_1 - \tau_2|} - n_b(\vec{k}) e^{\omega(\vec{k})|\tau_1 - \tau_2|}] \quad (124)$$

where

$$\bar{n}_b(\vec{k}) = (1 - e^{-\beta\varepsilon(\vec{k})})^{-1}, \quad n_b(\vec{k}) = (1 - e^{\beta\varepsilon(\vec{k})})^{-1} \quad (125)$$

are just the boson distribution functions [20-23, 29].

Let us turn to the calculation of the functional integral in Eq. (101). Based on the stationary-phase method, we can write

$$Z_q^0[\eta] = \int D(b^* b d^* d) \exp\{-\int d^3 k [b_s^*(\vec{k}) b_s(\vec{k}) + d_s^*(\vec{k}) d_s(\vec{k})]\} \times \exp\{I_q^0(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s)\} \quad (126)$$

where $I_q^0(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s)$ will be obtained from Eq. (102) or Eq. (104) by the stationary condition $\delta I_q(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s) = 0$. From this condition and the boundary conditions [21-26]:

$$\begin{aligned} b_s^*(\vec{k}, \beta) &= -b_s^*(\vec{k}), \quad b_s(\vec{k}, 0) = b_s(\vec{k}), \\ d_s^*(\vec{k}, \beta) &= -d_s^*(\vec{k}), \quad d_s(\vec{k}, 0) = d_s(\vec{k}), \end{aligned} \quad (127)$$

one may deduce from Eq. (102) or Eq. (104) the following equations [23-26]:

$$\begin{aligned} \dot{b}_s(\vec{k}, \tau) + \varepsilon(\vec{k}) b_s(\vec{k}, \tau) &= \eta_s(\vec{k}, \tau), \\ \dot{b}_s^*(\vec{k}, \tau) - \varepsilon(\vec{k}) b_s^*(\vec{k}, \tau) &= -\eta_s^*(\vec{k}, \tau), \\ \dot{d}_s(\vec{k}, \tau) + \varepsilon(\vec{k}) d_s(\vec{k}, \tau) &= \bar{\eta}_s(\vec{k}, \tau), \\ \dot{d}_s^*(\vec{k}, \tau) - \varepsilon(\vec{k}) d_s^*(\vec{k}, \tau) &= -\bar{\eta}_s^*(\vec{k}, \tau). \end{aligned} \quad (128)$$

Following the procedure described in Eqs. (111)-(116), the solutions to the above equations, which satisfies the boundary conditions in Eqs. (127) and the conditions like those in Eq. (113), can be found to be [23-26]

$$\begin{aligned} b_s(\vec{k}, \tau) &= b_s(\vec{k}) e^{-\varepsilon(\vec{k})\tau} + \int_0^\tau d\tau' e^{-\varepsilon(\vec{k})(\tau-\tau')} \eta_s(\vec{k}, \tau'), \\ b_s^*(\vec{k}, \tau) &= -b_s^*(\vec{k}) e^{\varepsilon(\vec{k})(\tau-\beta)} + \int_\tau^\beta d\tau' e^{\varepsilon(\vec{k})(\tau-\tau')} \eta_s^*(\vec{k}, \tau'), \\ d_s(\vec{k}, \tau) &= d_s(\vec{k}) e^{-\varepsilon(\vec{k})\tau} + \int_0^\tau d\tau' e^{-\varepsilon(\vec{k})(\tau-\tau')} \bar{\eta}_s(\vec{k}, \tau'), \\ d_s^*(\vec{k}, \tau) &= -d_s^*(\vec{k}) e^{\varepsilon(\vec{k})(\tau-\beta)} + \int_\tau^\beta d\tau' e^{\varepsilon(\vec{k})(\tau-\tau')} \bar{\eta}_s^*(\vec{k}, \tau'). \end{aligned} \quad (129)$$

Substituting the above solutions into Eq. (102) or (104), we find

$$\begin{aligned} I_q^0(b_s^*, b_s, d_s^*, d_s; \eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s) &= \int d^3 k \{-e^{-\beta\varepsilon(\vec{k})} [b_s^*(\vec{k}) b_s(\vec{k}) + d_s^*(\vec{k}) d_s(\vec{k})] \\ &+ \int_0^\beta d\tau e^{-\varepsilon(\vec{k})\tau} [\eta_s^*(\vec{k}, \tau) b_s(\vec{k}) + \bar{\eta}_s^*(\vec{k}, \tau) d_s(\vec{k})] - e^{-\beta\varepsilon(\vec{k})} \int_0^\beta d\tau e^{\varepsilon(\vec{k})\tau} \\ &\times [b_s^*(\vec{k}) \eta_s(\vec{k}, \tau) + d_s^*(\vec{k}) \bar{\eta}_s(\vec{k}, \tau)]\} + B[\eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s] \end{aligned} \quad (130)$$

where

$$\begin{aligned} B[\eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s] &= \frac{1}{2} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3 k \{\theta(\tau_1 - \tau_2) e^{-\varepsilon(\vec{k})(\tau_1 - \tau_2)} \\ &\times [\eta_s^*(\vec{k}, \tau_1) \eta_s(\vec{k}, \tau_2) + \bar{\eta}_s^*(\vec{k}, \tau_1) \bar{\eta}_s(\vec{k}, \tau_2)] + \theta(\tau_2 - \tau_1) e^{-\varepsilon(\vec{k})(\tau_2 - \tau_1)} \\ &\times [\eta_s^*(\vec{k}, \tau_2) \eta_s(\vec{k}, \tau_1) + \bar{\eta}_s^*(\vec{k}, \tau_2) \bar{\eta}_s(\vec{k}, \tau_1)]\} \end{aligned} \quad (131)$$

On inserting Eq. (130) into Eq. (126), we have

$$Z_q^0[\eta] = A[\eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s] e^{B[\eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s]} \quad (132)$$

where

$$\begin{aligned} A[\eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s] &= \int D(b^* b) \exp\{-\int d^3 k [b_s^*(\vec{k}) b_s(\vec{k}) (1 + e^{-\beta\varepsilon(\vec{k})}) \\ &+ e^{-\beta\varepsilon(\vec{k})} b_s^*(\vec{k})] \int_0^\beta d\tau e^{\varepsilon(\vec{k})\tau} \eta_s(\vec{k}, \tau) - \int_0^\beta d\tau e^{-\varepsilon(\vec{k})\tau} \eta_s^*(\vec{k}, \tau) b_s(\vec{k})]\} \\ &\times \int D(d^* d) \exp\{-\int d^3 k [d_s^*(\vec{k}) d_s(\vec{k}) (1 + e^{-\beta\varepsilon(\vec{k})}) + e^{-\beta\varepsilon(\vec{k})} d_s^*(\vec{k}) \\ &\times \int_0^\beta d\tau e^{\varepsilon(\vec{k})\tau} \bar{\eta}_s(\vec{k}, \tau) - \int_0^\beta d\tau e^{-\varepsilon(\vec{k})\tau} \bar{\eta}_s^*(\vec{k}, \tau) d_s(\vec{k})]\} \end{aligned} \quad (133)$$

here the fact that the two integrals over $\{b^*, b\}$ and $\{d^*, d\}$ commute with each other has been noted. Obviously, each of the above integrals can easily be calculated by applying the integration formulas shown in Eq. (39). The result is

$$A[\eta_s^*, \eta_s, \bar{\eta}_s^*, \bar{\eta}_s] = Z_q^0 \exp\left\{-\frac{1}{2} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k (1 + e^{\beta\varepsilon(\vec{k})})^{-1} \times \{e^{-\varepsilon(\vec{k})(\tau_1 - \tau_2)} [\eta_s^*(\vec{k}, \tau_1) \eta_s(\vec{k}, \tau_2) + \bar{\eta}_s^*(\vec{k}, \tau_1) \bar{\eta}_s(\vec{k}, \tau_2)] + e^{-\varepsilon(\vec{k})(\tau_2 - \tau_1)} [\eta_s^*(\vec{k}, \tau_2) \eta_s(\vec{k}, \tau_1) + \bar{\eta}_s^*(\vec{k}, \tau_2) \bar{\eta}_s(\vec{k}, \tau_1)]\}\right\} \quad (134)$$

where

$$Z_q^0 = \prod_{\vec{k}_s} [1 + e^{-\beta\varepsilon(\vec{k})}]^2 \quad (135)$$

which just is the partition function contributed from free quarks and antiquarks [21-23, 29]. It is noted that the two terms in the exponent of Eq. (134) are equal to one another as seen from the interchange of the integration variables τ_1 and τ_2 . After Eqs. (131) and (134) are substituted in Eq. (132), we get

$$Z_q^0[\eta] = Z_q^0 \exp\left\{\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k \{[\theta(\tau_1 - \tau_2) - (1 + e^{\beta\varepsilon(\vec{k})})^{-1}] e^{-\varepsilon(\vec{k})(\tau_1 - \tau_2)} \times [\eta_s^*(\vec{k}, \tau_1) \eta_s(\vec{k}, \tau_2) + \bar{\eta}_s^*(\vec{k}, \tau_1) \bar{\eta}_s(\vec{k}, \tau_2)] + [\theta(\tau_2 - \tau_1) - (1 + e^{\beta\varepsilon(\vec{k})})^{-1}] \times e^{-\varepsilon(\vec{k})(\tau_2 - \tau_1)} [\eta_s^*(\vec{k}, \tau_2) \eta_s(\vec{k}, \tau_1) + \bar{\eta}_s^*(\vec{k}, \tau_2) \bar{\eta}_s(\vec{k}, \tau_1)]\}\right\}. \quad (136)$$

When we interchange the variables τ_1 and τ_2 and set $\vec{k} \rightarrow -\vec{k}$ in the second term of the above integrals and notice the relation

$$\bar{\eta}_s^*(\vec{k}, \tau_2) \bar{\eta}_s(\vec{k}, \tau_1) = \eta_s^*(-\vec{k}, \tau_1) \eta_s(-\vec{k}, \tau_2) \quad (137)$$

which will be proved in the Appendix, the functional $Z_q^0[\eta]$ will eventually be represented as

$$Z_q^0[\eta] = Z_q^0 \exp\left\{\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k [\eta_s^*(\vec{k}, \tau_1) \Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2) \eta_{s'}(\vec{k}, \tau_2) + \bar{\eta}_s^*(\vec{k}, \tau_1) \Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2) \bar{\eta}_{s'}(\vec{k}, \tau_2)]\right\} \quad (138)$$

where

$$\Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2) = \delta^{ss'} \Delta_q(\vec{k}, \tau_1 - \tau_2) \quad (139)$$

with

$$\Delta_q(\vec{k}, \tau_1 - \tau_2) = \frac{1}{2} [\bar{n}_f(\vec{k}) e^{-\varepsilon(\vec{k})|\tau_1 - \tau_2|} - n_f(\vec{k}) e^{\varepsilon(\vec{k})|\tau_1 - \tau_2|}] \quad (140)$$

is the free quark (antiquark) propagator. In the above,

$$\bar{n}_f(\vec{k}) = (1 + e^{-\beta\varepsilon(\vec{k})})^{-1}, \quad n_f(\vec{k}) = (1 + e^{\beta\varepsilon(\vec{k})})^{-1} \quad (141)$$

are the fermion distribution functions [21-23, 29].

Finally,

let us calculate the generating functional $Z_c^0[\zeta]$. From the stationary condition $\delta I_c(c_a^*, c_a, \bar{c}_a^*, \bar{c}_a; \zeta_a^*, \zeta_a, \bar{\zeta}_a^*, \bar{\zeta}_a) = 0$ and the boundary conditions:

$$\bar{c}_a^*(\vec{k}, \beta) = \bar{c}_a^*(\vec{k}), \quad c_a^*(\vec{k}, \beta) = c_a^*(\vec{k}), \quad \bar{c}_a(\vec{k}, 0) = \bar{c}_a(\vec{k}), \quad c_a(\vec{k}, 0) = c_a(\vec{k}), \quad (142)$$

which are the same as those for scalar fields other than for the fermion fields [21], it is easy to derive from Eq. (106) or (108) the following equations of motion:

$$\begin{aligned} \dot{c}_a(\vec{k}, \tau) + \omega(\vec{k}) c_a(\vec{k}, \tau) &= -\zeta_a(\vec{k}, \tau), \\ \dot{c}_a^*(\vec{k}, \tau) - \omega(\vec{k}) c_a^*(\vec{k}, \tau) &= \zeta_a^*(\vec{k}, \tau), \\ \dot{\bar{c}}_a(\vec{k}, \tau) + \omega(\vec{k}) \bar{c}_a(\vec{k}, \tau) &= \bar{\zeta}_a(\vec{k}, \tau), \\ \dot{\bar{c}}_a^*(\vec{k}, \tau) - \omega(\vec{k}) \bar{c}_a^*(\vec{k}, \tau) &= -\bar{\zeta}_a^*(\vec{k}, \tau). \end{aligned} \quad (143)$$

By the same procedure as stated in Eqs. (111)-(116), the solutions to the above equations can be found to be

$$\begin{aligned} c_a(\vec{k}, \tau) &= c_a(\vec{k})e^{-\omega(\vec{k})\tau} - \int_0^\tau d\tau' e^{-\omega(\vec{k})(\tau-\tau')} \zeta_a(\vec{k}, \tau'), \\ c_a^*(\vec{k}, \tau) &= c_a^*(\vec{k})e^{\omega(\vec{k})(\tau-\beta)} - \int_\tau^\beta d\tau' e^{\omega(\vec{k})(\tau-\tau')} \zeta_a^*(\vec{k}, \tau'), \\ \bar{c}_a(\vec{k}, \tau) &= \bar{c}_a(\vec{k})e^{-\omega(\vec{k})\tau} + \int_0^\tau d\tau' e^{-\omega(\vec{k})(\tau-\tau')} \bar{\zeta}_a(\vec{k}, \tau'), \\ \bar{c}_a^*(\vec{k}, \tau) &= \bar{c}_a^*(\vec{k})e^{\omega(\vec{k})(\tau-\beta)} + \int_\tau^\beta d\tau' e^{\omega(\vec{k})(\tau-\tau')} \bar{\zeta}_a^*(\vec{k}, \tau'). \end{aligned} \quad (144)$$

Upon substituting the above solutions into Eq. (106) or Eq. (108), we find

$$\begin{aligned} I_c^0(c_a^*, c_a, \bar{c}_a^*, \bar{c}_a; \zeta_a^*, \zeta_a, \bar{\zeta}_a^*, \bar{\zeta}_a) &= \int d^3k \{ e^{-\beta\omega(\vec{k})} [\bar{c}_a^*(\vec{k})\bar{c}_a(\vec{k}) - c_a^*(\vec{k})c_a(\vec{k})] \\ &\quad + e^{-\beta\omega(\vec{k})} \int_0^\beta d\tau e^{\omega(\vec{k})\tau} [c_a^*(\vec{k})\zeta_a(\vec{k}, \tau) + \bar{c}_a^*(\vec{k})\bar{\zeta}_a(\vec{k}, \tau)] \\ &\quad - \int_0^\beta d\tau e^{-\omega(\vec{k})\tau} [\zeta_a^*(\vec{k}, \tau)c_a(\vec{k}) + \bar{\zeta}_a^*(\vec{k}, \tau)\bar{c}_a(\vec{k})] \} \\ &\quad - \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k \theta(\tau_1 - \tau_2) e^{-\omega(\vec{k})(\tau_1 - \tau_2)} [\zeta_a^*(\vec{k}, \tau_1)\zeta_a(\vec{k}, \tau_2) \\ &\quad - \bar{\zeta}_a^*(\vec{k}, \tau_1)\bar{\zeta}_a(\vec{k}, \tau_2)]. \end{aligned} \quad (145)$$

On inserting the above expression into the following integral given by the stationary-phase method:

$$Z_c^0[\zeta] = \int D(\bar{c}^* \bar{c} c c^*) \exp\left\{-\int d^3k [\bar{c}_a^*(\vec{k})\bar{c}_a(\vec{k}) - c_a^*(\vec{k})c_a(\vec{k})] + I_c^0(c_a^*, c_a, \bar{c}_a^*, \bar{c}_a; \zeta_a^*, \zeta_a, \bar{\zeta}_a^*, \bar{\zeta}_a)\right\}, \quad (146)$$

and applying the integration formulas in Eq. (39), one can get

$$\begin{aligned} Z_c^0[\zeta] &= Z_c^0 \exp\left\{\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k [\theta(\tau_1 - \tau_2) - (1 - e^{\beta\omega(\vec{k})})^{-1}] e^{-\omega(\vec{k})(\tau_1 - \tau_2)} \right. \\ &\quad \left. \times [\bar{\zeta}_a^*(\vec{k}, \tau_1)\bar{\zeta}_a(\vec{k}, \tau_2) - \zeta_a^*(\vec{k}, \tau_1)\zeta_a(\vec{k}, \tau_2)]\right\} \end{aligned} \quad (147)$$

where

$$Z_c^0 = \prod_{\vec{k}_a} [1 - e^{-\beta\omega(\vec{k})}]^2 \quad (148)$$

is just the partition function arising from the free ghost particles which plays the role of cancelling out the unphysical contribution contained in Eq. (120). If we change the integration variables in Eq. (147) and considering the relations

$$\zeta_a^*(\vec{k}, \tau) = -\bar{\zeta}_a(-\vec{k}, \tau), \quad \zeta_a(\vec{k}, \tau) = -\bar{\zeta}_a^*(-\vec{k}, \tau) \quad (149)$$

which will be interpreted in the Appendix, Eq. (147) may be recast in the form

$$\begin{aligned} Z_c^0[\zeta] &= Z_c^0 \exp\left\{\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3k [\bar{\zeta}_a^*(\vec{k}, \tau_1)\Delta_c^{aa'}(\vec{k}, \tau_1 - \tau_2)\bar{\zeta}_{a'}(\vec{k}, \tau_2) \right. \\ &\quad \left. - \zeta_a^*(\vec{k}, \tau_1)\Delta_c^{aa'}(\vec{k}, \tau_1 - \tau_2)\zeta_{a'}(\vec{k}, \tau_2)]\right\} \end{aligned} \quad (150)$$

where

$$\Delta_c^{aa'}(\vec{k}, \tau_1 - \tau_2) = \delta^{aa'} \Delta_g(\vec{k}, \tau_1 - \tau_2) \quad (151)$$

here $\Delta_g(\vec{k}, \tau_1 - \tau_2)$ was written in Eq. (124).

Up to the present, the perturbative expansion of the thermal QCD generating functional in the coherent-state representation has exactly been given by the combination of Eqs. (95), (96), (119), (138) and (150). In the derivation of the perturbation expansion, as one has seen, to obtain the final expressions of the propagators shown in Eqs. (124), (140) and (151), it is necessary to use the functional properties and relations for the external sources as denoted in Eqs. (123), (137) and (149). As a result of the derivation of the generating functional, the partition function for the free system has simultaneously been given by the combination of Eqs. (120), (135) and (148). The partition function for the interacting system can be calculated in the way as shown in Eq. (45). Here it should be noted that the differential $\delta/\delta j(\tau)$ in Eq. (95) represent the collection of the differentials $\delta/\delta \xi_\lambda^{a*}(\vec{k}, \tau)$, $\delta/\delta \xi_\lambda^a(\vec{k}, \tau)$, $-\delta/\delta \eta_s(\vec{k}, \tau)$, $\delta/\delta \eta_s^*(\vec{k}, \tau)$, $-\delta/\delta \bar{\eta}_s(\vec{k}, \tau)$, $\delta/\delta \bar{\eta}_s^*(\vec{k}, \tau)$, $-\delta/\delta \zeta_a(\vec{k}, \tau)$, $\delta/\delta \zeta_a^*(\vec{k}, \tau)$, $-\delta/\delta \bar{\zeta}_a(\vec{k}, \tau)$ and $\delta/\delta \bar{\zeta}_a^*(\vec{k}, \tau)$. Ordinarily, the generating functional of thermal QCD represented in the position space is used in the literature. In the Appendix, it will be shown that this generating functional can readily be derived from the generating functional described in this section.

V. RELATIVISTIC EQUATION FOR $Q\bar{Q}$ BOUND STATES

With the generating functional given in the preceding section, we are ready to derive the relativistic equation for $q\bar{q}$ bound states at finite temperature. It is well-known that a bound state exists in the space-like Minkowski space in which there always is an equal-time Lorentz frame. Since in the equal-time frame, the relativistic equation is reduced to a three-dimensional one without loss of any rigorism, in this section we only pay our attention to the three-dimensional equation which may be derived from the equations of motion satisfied by the following $q\bar{q}$ two-"time" (temperature) four-point Green function [15-17]

$$\begin{aligned}\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2) &= Tr\{e^{\beta(\Omega - \hat{K})} T\{N[\hat{b}_\alpha(\tau_1)\hat{b}_\beta^+(\tau_1)]N[\hat{b}_\gamma(\tau_2)\hat{b}_\delta^+(\tau_2)]\}\} \\ &\equiv \left\langle T\{N[\hat{b}_\alpha(\tau_1)\hat{b}_\beta^+(\tau_1)]N[\hat{b}_\gamma(\tau_2)\hat{b}_\delta^+(\tau_2)]\} \right\rangle_\beta\end{aligned}\quad (152)$$

where the symbol $\langle \cdots \rangle_\beta$ represents the statistical average and N symbolizes the normal product whose definition can be given from the corresponding definition at zero-temperature by replacing the vacuum average with the statistical average [17]

$$N[\hat{b}_\alpha(\tau_1)\hat{b}_\beta^+(\tau_2)] = T[\hat{b}_\alpha(\tau_1)\hat{b}_\beta^+(\tau_2)] - S_{\alpha\beta}(\tau_1 - \tau_2) \quad (153)$$

where

$$S_{\alpha\beta}(\tau_1 - \tau_2) = \left\langle T[\hat{b}_\alpha(\tau_1)\hat{b}_\beta^+(\tau_2)] \right\rangle_\beta \quad (154)$$

is the quark or antiquark thermal propagator. The normal product in Eq. (152) plays a role of excluding the contraction between the quark and the antiquark operators from the Green function when the quark and antiquark are of the same flavor. Physically, this avoids the $q\bar{q}$ annihilation that would break stability of a bound state. Substituting Eq. (153) in Eq. (152), we have

$$\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2) = G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2) - S_{\alpha\beta}S_{\gamma\delta} \quad (155)$$

where

$$G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2) = \left\langle T\{\hat{b}_\alpha(\tau_1)\hat{b}_\beta^+(\tau_1)\hat{b}_\gamma(\tau_2)\hat{b}_\delta^+(\tau_2)\} \right\rangle_\beta \quad (156)$$

is the ordinary Green function and, $S_{\alpha\beta}$ and $S_{\gamma\delta}$ are the equal-time quark (antiquark) propagators. Obviously, in order to derive the equation of motion satisfied by the Green function $\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)$, we need first to derive the equation of motion for the Green function $G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)$.

Let us start with the generating functional in Eq. (91). As shown in Sect. 4, by partial integration of the second term on the right hand side of Eq. (89), the generating functional may be written in the form

$$\begin{aligned}Z[j] &= \int D(q^*q) e^{-q^* \cdot q} \int \mathfrak{D}(q^*q) \exp\{q^*(\beta) \cdot q(\beta) \\ &\quad - S_E + \int_0^\beta d\tau j^*(\tau) \cdot q(\tau)\}\end{aligned}\quad (157)$$

where

$$S_E = \int_0^\beta d\tau \left\{ \sum_\alpha q_\alpha^*(\tau) \circ \dot{q}_\alpha(\tau) + H(\tau) \right\} \quad (158)$$

here $H(\tau)$ was given in Eqs. (78) and (79). First, we derive an equation of motion describing the variation of the $q\bar{q}$ four-point Green function with the "time" variable τ_1 . For this purpose, let us differentiate the generating functional in Eq. (157) with respect to $b_\alpha^*(\tau_1)$. Considering that the generating functional is independent of $b_\alpha^*(\tau_1)$ and noticing the expressions given in Eqs. (158), (78), (79) and (93), one may obtain

$$\begin{aligned}\frac{\delta Z[j]}{\delta b_\alpha^*(\tau_1)} &= \int D(q^*q) e^{-q^* \cdot q} \int \mathfrak{D}(q^*q) [-\dot{b}_\alpha(\tau_1) - \varepsilon_\alpha \theta_\alpha b_\alpha(\tau_1) \\ &\quad - \sum_{\rho\lambda} A_{\alpha\rho\lambda} b_\rho(\tau_1) a_\lambda(\tau_1) + \theta_\alpha \eta_\alpha(\tau_1)] \exp\{q^*(\beta) \cdot q(\beta) - S_E \\ &\quad - \int_0^\beta d\tau j^*(\tau) \cdot q(\tau)\} = 0.\end{aligned}\quad (159)$$

When the $b_\alpha(\tau_1)$ and $a_\lambda(\tau_1)$ in the above are replaced by the functional derivatives $\theta_\alpha \delta / \delta \eta_\alpha^*(\tau_1)$ and $\delta / \delta j_\lambda^*(\tau_1)$ respectively and multiplying the both sides of Eq. (159) with θ_α , the above equation can be written as

$$\left\{ \frac{d}{d\tau_1} \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} + \theta_\alpha \varepsilon_\alpha \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} + \sum_{\rho\lambda} \theta_\alpha \theta_\rho A(\alpha\rho\lambda) \frac{\delta^2}{\delta \eta_\rho^*(\tau_1) \delta j_\lambda^*(\tau_1)} - \eta_\alpha(\tau_1) \right\} Z[j] = 0. \quad (160)$$

Then, we differentiate the above equation with respect to the sources $\eta_\beta(\tau_1)$, giving

$$\left\{ \left(\frac{d}{d\tau_1} \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} \right) \frac{\delta}{\delta \eta_\beta(\tau_1)} + \theta_\alpha \varepsilon_\alpha \frac{\delta^2}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_1)} + \sum_{\rho\lambda} \theta_\alpha \theta_\rho A(\alpha\rho\lambda) \frac{\delta^3}{\delta \eta_\rho^*(\tau_1) \delta \eta_\beta(\tau_1) \delta j_\lambda^*(\tau_1)} + \delta_{\alpha\beta} - \eta_\alpha(\tau_1) \frac{\delta}{\delta \eta_\beta(\tau_1)} \right\} Z[j] = 0. \quad (161)$$

Furthermore, successive differentiations of Eq. (161) with respect sources $\eta_\gamma^*(\tau_2)$ and $\eta_\delta(\tau_2)$ yield

$$\begin{aligned} & \left\{ \left(\frac{d}{d\tau_1} \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} \right) \frac{\delta^3}{\delta \eta_\beta(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} + \theta_\alpha \varepsilon_\alpha \frac{\delta^4}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \right. \\ & + \sum_{\lambda\sigma} \theta_\alpha \theta_\rho A(\alpha\rho\lambda) \frac{\delta^5}{\delta \eta_\rho^*(\tau_1) \delta \eta_\beta(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2) \delta j_\lambda^*(\tau_1)} + \delta_{\alpha\beta} \frac{\delta^2}{\delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \\ & \left. - \delta_{\alpha\delta} \delta(\tau_1 - \tau_2) \frac{\delta^2}{\delta \eta_\gamma^*(\tau_2) \delta \eta_\beta(\tau_1)} - \eta_\alpha(\tau_1) \frac{\delta^3}{\delta \eta_\beta(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \right\} Z[j] = 0. \end{aligned} \quad (162)$$

Similarly, when differentiating Eq. (157) with respect $b_\beta(\tau_1)$, one may obtain

$$\left\{ \frac{d}{d\tau_1} \frac{\delta}{\delta \eta_\beta(\tau_1)} - \theta_\beta \varepsilon_\beta \frac{\delta}{\delta \eta_\beta(\tau_1)} - \sum_{\sigma\lambda} \theta_\sigma \theta_\beta A(\sigma\beta\lambda) \frac{\delta^2}{\delta \eta_\sigma(\tau_1) \delta j_\lambda^*(\tau_1)} - \eta_\beta^*(\tau_1) \right\} Z[j] = 0, \quad (163)$$

Subsequently, On differentiating the above equation with respect to $\eta_\alpha^*(\tau_1)$, we get

$$\left\{ \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} \left(\frac{d}{d\tau_1} \frac{\delta}{\delta \eta_\beta(\tau_1)} \right) - \theta_\beta \varepsilon_\beta \frac{\delta^2}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_1)} - \sum_{\sigma\lambda} \theta_\beta \theta_\sigma A(\sigma\beta\lambda) \frac{\delta^3}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\sigma(\tau_1) \delta j_\lambda^*(\tau_1)} - \delta_{\alpha\beta} + \eta_\beta^*(\tau_1) \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} \right\} Z[j] = 0. \quad (164)$$

Finally, successive differentiations of the above equation with respect to the sources $\eta_\gamma^*(\tau_2)$ and $\eta_\delta(\tau_2)$ give rise to

$$\begin{aligned} & \left\{ \frac{\delta}{\delta \eta_\alpha^*(\tau_1)} \left(\frac{d}{d\tau_1} \frac{\delta}{\delta \eta_\beta(\tau_1)} \right) \frac{\delta^2}{\delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} - \theta_\beta \varepsilon_\beta \frac{\delta^4}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \right. \\ & - \sum_{\lambda\sigma} \theta_\beta \theta_\sigma A(\sigma\beta\lambda) \frac{\delta^5}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\sigma(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2) \delta j_\lambda^*(\tau_1)} - \delta_{\alpha\beta} \frac{\delta^2}{\delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \\ & \left. + \delta_{\beta\gamma} \delta(\tau_1 - \tau_2) \frac{\delta^2}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\delta(\tau_2)} + \eta_\beta^*(\tau_1) \frac{\delta^3}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \right\} Z[j] = 0. \end{aligned} \quad (165)$$

Adding Eq. (161) to Eq. (164), then multiplying the both sides of the equation thus obtained with $-\theta_\alpha \theta_\beta$ and finally setting the external sources $\eta_\alpha^* = \eta_\beta = 0$, but remaining the gluon source $j_\lambda \neq 0$, we get

$$\left(\frac{d}{d\tau_1} + \theta_\alpha \varepsilon_\alpha - \theta_\beta \varepsilon_\beta \right) S_{\alpha\beta}^{j_\lambda} + \sum_{\rho\sigma\lambda} [A(\alpha\rho\lambda) \delta_{\beta\sigma} - A(\sigma\beta\lambda) \delta_{\alpha\rho}] \frac{\delta}{\delta j_\lambda^*(\tau_1)} S_{\rho\sigma}^{j_\lambda} = 0 \quad (166)$$

where

$$S_{\alpha\beta}^{j_\lambda} = -\frac{1}{Z} \theta_\alpha \theta_\beta \frac{\delta^2 Z[j]}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_1)} \Big|_{\eta_\alpha^* = \eta_\beta = 0} \quad (167)$$

is the quark (antiquark) equal-time propagator in the presence of source j_λ . If we define

$$H(\alpha\beta; \rho\sigma; \tau_1)^{j_\lambda} = \left(\frac{d}{d\tau_1} + \theta_\alpha \varepsilon_\alpha - \theta_\beta \varepsilon_\beta \right) \delta_{\alpha\rho} \delta_{\beta\sigma} + \sum_\lambda f(\alpha\beta; \rho\sigma\lambda) \frac{\delta}{\delta j_\lambda^*(\tau_1)} \quad (168)$$

where

$$f(\alpha\beta; \rho\sigma\lambda) = A(\alpha\rho\lambda) \delta_{\beta\sigma} - A(\sigma\beta\lambda) \delta_{\alpha\rho}, \quad (169)$$

Eq. (166) can simply be represented as

$$\sum_{\rho\sigma} H(\alpha\beta; \rho\sigma; \tau_1)^{j\lambda} S_{\rho\sigma}^{j\lambda} = 0. \quad (170)$$

When summing up the both equations in Eqs. (162) and (165), then multiplying the equation thus obtained with $\theta_\alpha\theta_\beta\theta_\gamma\theta_\delta$ and finally setting all the sources but j_λ to be zero, one may get

$$\begin{aligned} & \left(\frac{d}{d\tau_1} + \theta_\alpha\varepsilon_\alpha - \theta_\beta\varepsilon_\beta\right)G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} + \sum_{\rho\sigma\lambda} f(\alpha\beta; \rho\sigma\lambda) \frac{\delta}{\delta j_\lambda^*(\tau_1)} G(\rho\sigma; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} \\ & = \delta(\tau_1 - \tau_2) [\delta_{\beta\gamma} S_{\alpha\delta}(\tau_1 - \tau_2)^{j\lambda} - \delta_{\alpha\delta} S_{\gamma\beta}(\tau_2 - \tau_1)^{j\lambda}] \end{aligned} \quad (171)$$

where

$$G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} = \frac{1}{Z} \theta_\alpha \theta_\beta \theta_\gamma \theta_\delta \frac{\delta^4 Z[j]}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_1) \delta \eta_\gamma^*(\tau_2) \delta \eta_\delta(\tau_2)} \Big|_{\eta^*=\eta=0} \quad (172)$$

and

$$S_{\alpha\beta}(\tau_1 - \tau_2)^{j\lambda} = -\frac{1}{Z} \theta_\alpha \theta_\beta \frac{\delta^2 Z[j]}{\delta \eta_\alpha^*(\tau_1) \delta \eta_\beta(\tau_2)} \Big|_{\eta^*=\eta=0} \quad (173)$$

are respectively the $q\bar{q}$ two-”time” four-point thermal Green function and the quark or antiquark thermal propagator in presence of source j_λ . When the source j_λ is turned off, Eqs. (172) and (173) will respectively go over to the Green function in Eq. (156) and the propagator in Eq. (154). It is noted that due to the restriction of the delta function, the propagators in Eq. (171) are actually ”time”-independent. With the definition in Eq. (168), Eq. (171) may be represented as

$$\sum_{\rho\sigma} H(\alpha\beta; \rho\sigma; \tau_1)^{j\lambda} G(\rho\sigma; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} = -\delta(\tau_1 - \tau_2) S(\alpha\beta; \gamma\delta)^{j\lambda} \quad (174)$$

where

$$S(\alpha\beta; \gamma\delta)^{j\lambda} = \delta_{\alpha\delta} S_{\gamma\beta}^{j\lambda} - \delta_{\beta\gamma} S_{\alpha\delta}^{j\lambda}. \quad (175)$$

Acting on the both sides of Eq. (155) with the operator $H(\alpha\beta; \rho\sigma; \tau_1)^{j\lambda}$ and using the equations in Eqs. (170) and (174), we find

$$\begin{aligned} \sum_{\rho\sigma} H(\alpha\beta; \rho\sigma; \tau_1)^{j\lambda} \mathcal{G}(\rho\sigma; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} &= \sum_{\rho\sigma} H(\alpha\beta; \rho\sigma; \tau_1)^{j\lambda} G(\rho\sigma; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} \\ &= -\delta(\tau_1 - \tau_2) S(\alpha\beta; \gamma\delta)^{j\lambda} \end{aligned} \quad (176)$$

This indicates that the equation of motion satisfied by the Green function $\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)$ formally is the same as the one shown in Eq. (171). Therefore, in the case that the source j_λ vanishes, we can write

$$\begin{aligned} & \left(\frac{d}{d\tau_1} + \theta_\alpha\varepsilon_\alpha - \theta_\beta\varepsilon_\beta\right)\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2) \\ & = -\delta(\tau_1 - \tau_2) S(\alpha\beta; \gamma\delta) - \sum_{\rho\sigma\lambda} f(\alpha\beta; \rho\sigma\lambda) \mathcal{G}(\rho\sigma\lambda; \gamma\delta; \tau_1 - \tau_2) \end{aligned} \quad (177)$$

where

$$\begin{aligned} \mathcal{G}(\rho\sigma\lambda; \gamma\delta; \tau_1 - \tau_2) &= \frac{\delta}{\delta j_\lambda^*(\tau_1)} \mathcal{G}(\rho\sigma; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} \Big|_{j_\lambda=0} \\ &= \left\langle T \{ N[\hat{b}_\rho(\tau_1) \hat{b}_\sigma^+(\tau_1) \hat{a}_\lambda(\tau_1)] N[\hat{b}_\gamma(\tau_2) \hat{b}_\delta^+(\tau_2)] \} \right\rangle_\beta \end{aligned} \quad (178)$$

and

$$S(\alpha\beta; \gamma\delta) = \delta_{\alpha\delta} S_{\gamma\beta} - \delta_{\beta\gamma} S_{\alpha\delta} = -\langle [\hat{b}_\alpha \hat{b}_\beta^+, \hat{b}_\gamma \hat{b}_\delta^+]_- \rangle_\beta. \quad (179)$$

It is noted here that similar to the definition in Eq. (153), the normal product $N[\hat{b}_\rho(\tau_1) \hat{b}_\sigma^+(\tau_1) \hat{a}_\lambda(\tau_1)]$ in Eq. (178) is defined as

$$N[\hat{b}_\rho(\tau_1) \hat{b}_\sigma^+(\tau_1) \hat{a}_\lambda(\tau_1)] = T[\hat{b}_\rho(\tau_1) \hat{b}_\sigma^+(\tau_1) \hat{a}_\lambda(\tau_1)] - \Lambda(\rho\sigma\lambda) \quad (180)$$

where

$$\Lambda(\rho\sigma\lambda) = \left\langle T[\widehat{b}_\rho(\tau_1)\widehat{b}_\sigma^+(\tau_1)\widehat{a}_\lambda(\tau_1)] \right\rangle_\beta. \quad (181)$$

Substituting Eqs. (153) and (180) into Eq. (178), we have

$$\mathcal{G}(\rho\sigma\lambda; \gamma\delta; \tau_1 - \tau_2) = G(\rho\sigma\lambda; \gamma\delta; \tau_1 - \tau_2) - \Lambda(\rho\sigma\lambda)S_{\gamma\delta} \quad (182)$$

where

$$G(\rho\sigma\lambda; \gamma\delta; \tau_1 - \tau_2) = \left\langle T\{\widehat{b}_\rho(\tau_1)\widehat{b}_\sigma^+(\tau_1)\widehat{a}_\lambda(\tau_1)\widehat{b}_\gamma(\tau_2)\widehat{b}_\delta^+(\tau_2)\} \right\rangle_\beta \quad (183)$$

is the ordinary five-point thermal Green function including a gluon operator in it.

By the argument as mentioned in the Appendix (see Eq. (A9) or (A18)), it is easy to prove that the Green functions $\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)$ and $\mathcal{G}(\rho\sigma\lambda; \gamma\delta; \tau_1 - \tau_2)$ are periodic. Therefore, we have the following Fourier expansions:

$$\begin{aligned} \mathcal{G}(\alpha\beta; \gamma\delta; \tau) &= \frac{1}{\beta} \sum_n \mathcal{G}(\alpha\beta; \gamma\delta; \omega_n) e^{-i\omega_n \tau}, \\ \mathcal{G}(\rho\sigma\lambda; \gamma\delta; \tau) &= \frac{1}{\beta} \sum_n \mathcal{G}(\rho\sigma\lambda; \gamma\delta; \omega_n) e^{-i\omega_n \tau} \end{aligned} \quad (184)$$

where $\tau = \tau_1 - \tau_2$ and $\omega_n = \frac{2\pi n}{\beta}$. Upon inserting Eq. (184) into Eq. (177) and performing the integration $\frac{1}{2} \int_{-\beta}^{\beta} d\tau e^{i\omega_n \tau}$, we arrive at

$$\begin{aligned} & (i\omega_n - \theta_\alpha \varepsilon_\alpha + \theta_\beta \varepsilon_\beta) \mathcal{G}(\alpha\beta; \gamma\delta; \omega_n) \\ &= -S(\alpha\beta; \gamma\delta) + \sum_{\rho\sigma\lambda} f(\alpha\beta; \rho\sigma\lambda) \mathcal{G}(\rho\sigma\lambda; \gamma\delta; \omega_n). \end{aligned} \quad (185)$$

It is well-known that the Green function $\mathcal{G}(\rho\sigma\lambda; \gamma\delta; \omega_n)$ is B-S (two-particle) reducible [15-17]. Therefore, we can write

$$\sum_{\lambda\tau\rho} f(\alpha\beta; \rho\sigma\lambda) \mathcal{G}(\rho\sigma\lambda; \gamma\delta; \omega_n) = \sum_{\mu\nu} K(\alpha\beta; \mu\nu; \omega_n) \mathcal{G}(\mu\nu; \gamma\delta; \omega_n) \quad (186)$$

where $K(\alpha\beta; \mu\nu; \omega_n)$ is called interaction kernel. Thus, Eq. (185) can be written in a closed form

$$(i\omega_n - \theta_\alpha \varepsilon_\alpha + \theta_\beta \varepsilon_\beta) \mathcal{G}(\alpha\beta; \gamma\delta; \omega_n) = -S(\alpha\beta; \gamma\delta) + \sum_{\mu\nu} K(\alpha\beta; \mu\nu; \omega_n) \mathcal{G}(\mu\nu; \gamma\delta; \omega_n). \quad (187)$$

Now, let us turn to the equation satisfied by $q\bar{q}$ bound states. This equation can be derived from Eq. (187) with the aid of the following Lehmann representation of the four-point Green function which may be derived by expanding the time-ordered product in Eq. (152) and then inserting the complete set of $q\bar{q}$ bound states into Eq. (152) [15-17, 31],

$$\mathcal{G}(\alpha\beta; \gamma\delta; \omega_l) = \frac{1}{2} e^{\beta\Omega} \sum_{mn} \Delta_{mn} \left\{ \frac{\chi_{nm}(\alpha\beta) \chi_{mn}(\gamma\delta)}{i\omega_l - E_{nm}} - \frac{\chi_{nm}(\gamma\delta) \chi_{mn}(\alpha\beta)}{i\omega_l + E_{nm}} \right\} \quad (188)$$

where

$$\chi_{nm}(\alpha\beta) = \left\langle m \left| N[\widehat{b}_\alpha \widehat{b}_\beta^+] \right| n \right\rangle \quad (189)$$

which is the transition amplitude from the state with energy E_n to the state with energy E_m and

$$\Delta_{nm} = e^{-\beta E_n} - e^{-\beta E_m}. \quad (190)$$

Upon substituting Eq. (188) into Eq. (187) and then taking the limit: $\lim_{i\omega_l \rightarrow E_{nm}} (i\omega_l - E_{nm})$, we get the following equation satisfied by the transition amplitude

$$(E_{nm} - \theta_\alpha \varepsilon_\alpha + \theta_\beta \varepsilon_\beta) \chi_{nm}(\alpha\beta) = \sum_{\gamma\delta} K(\alpha\beta; \gamma\delta; E_{nm}) \chi_{nm}(\gamma\delta) \quad (191)$$

where the fact that the function $S(\alpha\beta; \gamma\delta)$ has no bound state poles has been considered. If we take $|m\rangle$ to be the vacuum state $|0\rangle$ and set $E = E_{n0}$ and $\chi_n(\alpha\beta) = \langle 0 | N[\widehat{b}_\alpha \widehat{b}_\beta^\dagger] | n \rangle$, we can write from the above equation that

$$(E - \theta_\alpha \varepsilon_\alpha + \theta_\beta \varepsilon_\beta) \chi_n(\alpha\beta) = \sum_{\gamma\delta} K(\alpha\beta; \gamma\delta; E) \chi_n(\gamma\delta). \quad (192)$$

where the subscript n in E_n has been suppressed. This just is the equation satisfied by the $q\bar{q}$ bound states at finite temperature.

Since the index α contains $\theta_\alpha = \pm$, Eq. (192) actually is a set of coupled equations for the amplitudes $\chi_n(\alpha^+ \beta^-)$, $\chi_n(\alpha^- \beta^+)$, $\chi_n(\alpha^+ \beta^+)$ and $\chi_n(\alpha^- \beta^-)$. Following the procedure described in Refs. (16) and (17), one may reduce the above equation to an equivalent equation satisfied by the amplitude of positive energy. We do not repeat the derivation here. We only show the result as follows:

$$[E - \varepsilon(\vec{k}_\alpha) - \varepsilon(\vec{k}_\beta)] \psi(\alpha\beta; E) = \sum_{\gamma\delta} V(\alpha\beta; \gamma\delta; E) \psi(\gamma\delta; E). \quad (193)$$

where $\psi(\alpha\beta; E) = \chi_n(\alpha^+ \beta^-)$ and $V(\alpha\beta; \gamma\delta; E)$ is the interaction Hamiltonian which can be expressed as

$$V(\alpha\beta; \gamma\delta; E) = \sum_{n=0} V^{(n)}(\alpha\beta; \gamma\delta; E), \quad (194)$$

in which

$$V^{(0)}(\alpha\beta; \gamma\delta; E) = K_{++++}(\alpha\beta; \gamma\delta; E), \quad (195)$$

$$V^{(1)}(\alpha\beta; \gamma\delta; E) = \sum_{ab \neq ++} \sum_{\rho\sigma} \frac{K_{++ab}(\alpha\beta; \rho\sigma; E) K_{ab++}(\rho\sigma; \gamma\delta; E)}{E - a\varepsilon(\vec{k}_\rho) - b\varepsilon(\vec{k}_\sigma)}, \quad (196)$$

$$\begin{aligned} V^{(2)}(\alpha\beta; \gamma\delta; E) \\ = \sum_{ab \neq ++} \sum_{cd \neq ++} \sum_{\rho\sigma} \sum_{\mu\nu} \frac{K_{++ab}(\alpha\beta; \rho\sigma; E) K_{abcd}(\rho\sigma; \mu\nu; E) K_{cd++}(\mu\nu; \gamma\delta; E)}{(E - a\varepsilon(\vec{k}_\rho) - b\varepsilon(\vec{k}_\sigma))(E - c\varepsilon(\vec{k}_\mu) - d\varepsilon(\vec{k}_\nu))}, \\ \dots \dots \dots \end{aligned} \quad (197)$$

here $a, b = \pm$, and

$$\begin{aligned} K_{++++}(\alpha\beta; \gamma\delta; E) &= K(\alpha^+ \beta^-; \gamma^+ \delta^-; E), \\ K_{----}(\alpha\beta; \gamma\delta; E) &= K(\alpha^- \beta^+; \gamma^- \delta^+; E), \\ K_{+-+-}(\alpha\beta; \gamma\delta; E) &= K(\alpha^+ \beta^+; \gamma^+ \delta^+; E), \\ K_{-+-+}(\alpha\beta; \gamma\delta; E) &= K(\alpha^- \beta^-; \gamma^- \delta^-; E). \end{aligned} \quad (198)$$

VI. CLOSED EXPRESSION OF THE INTERACTION KERNEL IN THE EQUATION FOR $Q\bar{Q}$ BOUND STATES

In this section, we are devoted to deriving a closed expression of the interaction kernel appearing in Eq. (192) and defined in Eq. (186). For this derivation, we need equations of motion which describe evolution of the Green functions $\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)$ and $\mathcal{G}(\alpha\beta\sigma; \gamma\delta; \tau_1 - \tau_2)$ with time τ_2 . Taking the derivatives of the generating functional in Eq. (157) with respect to $b_\gamma^*(\tau_2)$ and $b_\delta(\tau_2)$ respectively, by the same procedure as described in the derivation of Eq. (160), one may obtain

$$\left\{ \frac{d}{d\tau_2} \frac{\delta}{\delta \eta_\gamma^*(\tau_2)} + \theta_\gamma \varepsilon_\gamma \frac{\delta}{\delta \eta_\gamma^*(\tau_2)} + \sum_{\rho\lambda} \theta_\gamma \theta_\rho A(\gamma\rho\lambda) \frac{\delta^2}{\delta \eta_\rho^*(\tau_2) \delta j_\lambda^*(\tau_2)} - \eta_\gamma(\tau_2) \right\} Z[j] = 0. \quad (199)$$

and

$$\left\{ \frac{d}{d\tau_2} \frac{\delta}{\delta\eta_\delta(\tau_2)} - \theta_\delta \varepsilon_\delta \frac{\delta}{\delta\eta_\delta(\tau_2)} - \sum_{\sigma\lambda} \theta_\delta \theta_\sigma A(\sigma\delta\lambda) \frac{\delta^2}{\delta\eta_\sigma(\tau_2) \delta j_\lambda^*(\tau_2)} - \eta_\delta^*(\tau_2) \right\} Z[j] = 0. \quad (200)$$

Performing differentiations of Eqs. (199) and (200) with respect to the sources $\eta_\delta(\tau_2)$ and $\eta_\gamma^*(\tau_2)$ respectively, we get

$$\begin{aligned} & \left\{ \left(\frac{d}{d\tau_2} \frac{\delta}{\delta\eta_\gamma^*(\tau_2)} \right) \frac{\delta}{\delta\eta_\delta(\tau_2)} + \theta_\gamma \varepsilon_\gamma \frac{\delta^2}{\delta\eta_\gamma^*(\tau_2) \delta\eta_\delta(\tau_2)} + \sum_{\rho\lambda} \theta_\gamma \theta_\rho A(\gamma\rho\lambda) \frac{\delta^3}{\delta\eta_\rho^*(\tau_2) \delta\eta_\delta(\tau_2) \delta j_\lambda^*(\tau_2)} \right. \\ & \quad \left. + \delta_{\gamma\delta} - \eta_\gamma(\tau_2) \frac{\delta}{\delta\eta_\delta(\tau_2)} \right\} Z[j] = 0 \end{aligned} \quad (201)$$

and

$$\begin{aligned} & \left\{ \frac{\delta}{\delta\eta_\gamma^*(\tau_2)} \left(\frac{d}{d\tau_2} \frac{\delta}{\delta\eta_\delta(\tau_2)} \right) - \theta_\delta \varepsilon_\delta \frac{\delta}{\delta\eta_\gamma^*(\tau_2) \delta\eta_\delta(\tau_2)} - \sum_{\sigma\lambda} \theta_\delta \theta_\sigma A(\sigma\delta\lambda) \frac{\delta^3}{\delta\eta_\gamma^*(\tau_2) \delta\eta_\sigma(\tau_2) \delta j_\lambda^*(\tau_2)} \right. \\ & \quad \left. - \delta_{\gamma\delta} + \eta_\delta^*(\tau_2) \frac{\delta}{\delta\eta_\gamma^*(\tau_2)} \right\} Z[j] = 0. \end{aligned} \quad (202)$$

Furthermore, by successively differentiating Eqs. (201) and (202) with respect to the sources $\eta_\alpha^*(\tau_1)$ and $\eta_\beta(\tau_1)$, one obtains

$$\begin{aligned} & \left\{ \frac{\delta^2}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1)} \left(\frac{d}{d\tau_2} \frac{\delta}{\delta\eta_\gamma^*(\tau_2)} \right) \frac{\delta}{\delta\eta_\delta(\tau_2)} + \theta_\gamma \varepsilon_\gamma \frac{\delta^4}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\gamma^*(\tau_2) \delta\eta_\delta(\tau_2)} \right. \\ & + \sum_{\lambda\sigma} \theta_\gamma \theta_\lambda A(\gamma\rho\lambda) \frac{\delta^5}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\rho^*(\tau_2) \delta\eta_\delta(\tau_2) \delta j_\lambda^*(\tau_2)} + \delta_{\gamma\delta} \frac{\delta^2}{\delta\eta_\alpha^*(\tau_2) \delta\eta_\beta(\tau_2)} \\ & \quad \left. - \delta_{\beta\gamma} \delta(\tau_1 - \tau_2) \frac{\delta^2}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\delta(\tau_2)} - \eta_\gamma(\tau_2) \frac{\delta^3}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\delta(\tau_2)} \right\} Z[j] = 0 \end{aligned} \quad (203)$$

and

$$\begin{aligned} & \left\{ \frac{\delta^3}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\gamma^*(\tau_2)} \left(\frac{d}{d\tau_2} \frac{\delta}{\delta\eta_\delta(\tau_2)} \right) - \theta_\delta \varepsilon_\delta \frac{\delta^4}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\gamma^*(\tau_2) \delta\eta_\delta(\tau_2)} \right. \\ & - \sum_{\sigma\lambda} \theta_\delta \theta_\sigma A(\sigma\delta\lambda) \frac{\delta^5}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\gamma^*(\tau_2) \delta\eta_\sigma(\tau_2) \delta j_\lambda^*(\tau_2)} - \delta_{\gamma\delta} \frac{\delta^2}{\delta\eta_\alpha^*(\tau_2) \delta\eta_\beta(\tau_2)} \\ & \quad \left. + \delta_{\alpha\delta} \delta(\tau_1 - \tau_2) \frac{\delta^2}{\delta\eta_\gamma^*(\tau_2) \delta\eta_\beta(\tau_1)} + \eta_\delta^*(\tau_2) \frac{\delta^3}{\delta\eta_\alpha^*(\tau_1) \delta\eta_\beta(\tau_1) \delta\eta_\gamma^*(\tau_2)} \right\} Z[j] = 0. \end{aligned} \quad (204)$$

Let us sum up Eqs. (201) and (202) at first, then multiply the both sides of the equation thus obtained with $-\theta_\gamma \theta_\delta$ and finally set all the sources but the source j_λ to vanish. By these operations, we get

$$\sum_{\rho\sigma} \overline{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} S_{\rho\sigma}^{j\lambda} = 0 \quad (205)$$

where

$$\overline{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} = \left(\frac{d}{d\tau_2} + \theta_\gamma \varepsilon_\gamma - \theta_\delta \varepsilon_\delta \right) \delta_{\gamma\rho} \delta_{\delta\sigma} - \sum_\lambda f(\rho\sigma\lambda; \gamma\delta) \frac{\delta}{\delta j_\lambda^*(\tau_2)} \quad (206)$$

in which

$$f(\rho\sigma\lambda; \gamma\delta) = A(\sigma\delta\lambda) \delta_{\gamma\rho} - A(\gamma\rho\lambda) \delta_{\delta\sigma} = -f(\gamma\delta; \rho\sigma\lambda) \quad (207)$$

and $S_{\rho\sigma}^{j\lambda}$ was defined in Eq. (167).

When we sum up Eqs. (203) and (204), then multiply the both sides of the equation thus obtained with $\theta_\alpha \theta_\beta \theta_\gamma \theta_\delta$ and finally set all the sources but the source j_λ to be zero, according to the definitions in Eqs. (172) and (173), it is found that

$$\sum_{\rho\sigma} \overline{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} = \delta(\tau_1 - \tau_2) [\delta_{\alpha\delta} S_{\gamma\beta}(\tau_2 - \tau_1)^{j\lambda} - \delta_{\beta\gamma} S_{\alpha\delta}(\tau_1 - \tau_2)^{j\lambda}]. \quad (208)$$

In order to derive the equation of motion satisfied by the Green function $G(\lambda\tau\sigma; \gamma\delta; \tau_1 - \tau_2)$ defined in Eq. (183), we may take the derivative of Eq. (208) with respect to $j_\lambda^*(\tau_1)$. In this way, we get

$$\sum_{\rho\sigma} \overline{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} G(\alpha\beta\lambda; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} = \delta(\tau_1 - \tau_2) [\delta_{\alpha\delta} \Lambda(\gamma\beta\rho; \tau_2 - \tau_1)^{j\lambda} - \delta_{\beta\gamma} \Lambda(\alpha\delta\rho; \tau_1 - \tau_2)^{j\lambda}] \quad (209)$$

where

$$\Lambda(\gamma\beta\rho; \tau_2 - \tau_1)^{j\lambda} = \frac{\delta}{\delta j_\lambda^*(\tau_1)} S_{\gamma\beta}(\tau_2 - \tau_1)^{j\lambda}, \quad (210)$$

$$\Lambda(\alpha\delta\rho; \tau_1 - \tau_2)^{j\lambda} = \frac{\delta}{\delta j_\lambda^*(\tau_1)} S_{\alpha\delta}(\tau_1 - \tau_2)^{j\lambda} \quad (211)$$

and

$$G(\alpha\beta\lambda; \gamma\delta; \tau_1 - \tau_2) = \frac{\delta}{\delta j_\lambda^*(\tau_1)} G(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)^{j\lambda}. \quad (212)$$

Acting on Eqs. (155) and (182) with the operator $\bar{H}(\gamma\delta; \rho\sigma; \tau_2)$ and employing Eq. (205), we find

$$\sum_{\rho\sigma} \bar{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} \mathcal{G}(\alpha\beta; \rho\sigma; \tau_1 - \tau_2)^{j\lambda} = \sum_{\rho\sigma} \bar{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} G(\alpha\beta; \rho\sigma; \tau_1 - \tau_2)^{j\lambda} \quad (213)$$

and

$$\sum_{\rho\sigma} \bar{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} \mathcal{G}(\alpha\beta\lambda; \rho\sigma; \tau_1 - \tau_2)^{j\lambda} = \sum_{\rho\sigma} \bar{H}(\gamma\delta; \rho\sigma; \tau_2)^{j\lambda} G(\alpha\beta\lambda; \rho\sigma; \tau_1 - \tau_2)^{j\lambda}. \quad (214)$$

The above two equalities further indicate that the equations of motion satisfied by the Green functions $\mathcal{G}(\alpha\beta; \rho\sigma; \tau_1 - \tau_2)^{j\lambda}$ and $\mathcal{G}(\alpha\beta\lambda; \rho\sigma; \tau_1 - \tau_2)^{j\lambda}$ are formally the same as those for the ordinary Green functions $G(\alpha\beta; \rho\sigma; \tau_1 - \tau_2)^{j\lambda}$ and $G(\alpha\beta\lambda; \rho\sigma; \tau_1 - \tau_2)^{j\lambda}$ respectively. Upon inserting Eqs. (208) into Eq. (213) and Eq. (209) into Eq. (214) and turning off the source j_λ , noticing the definition in Eq. (206), we derive the following equations

$$\left(\frac{d}{d\tau_2} + \theta_\gamma \varepsilon_\gamma - \theta_\delta \varepsilon_\delta \right) \mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2) = \delta(\tau_1 - \tau_2) [\delta_{\alpha\delta} S_{\gamma\beta}(\tau_2 - \tau_1) - \delta_{\beta\gamma} S_{\alpha\delta}(\tau_1 - \tau_2)] + \sum_{\lambda\tau\sigma} \mathcal{G}(\alpha\beta; \lambda\tau\sigma; \tau_1 - \tau_2) f(\lambda\tau\sigma; \gamma\delta) \quad (215)$$

and

$$\left(\frac{d}{d\tau_2} + \theta_\gamma \varepsilon_\gamma - \theta_\delta \varepsilon_\delta \right) \mathcal{G}(\alpha\beta\rho; \gamma\delta; \tau_1 - \tau_2) = \delta(\tau_1 - \tau_2) [\delta_{\alpha\delta} \Lambda(\gamma\beta\rho; \tau_2 - \tau_1) - \delta_{\beta\gamma} \Lambda(\alpha\delta\rho; \tau_1 - \tau_2)] + \sum_{\lambda\tau\sigma} \mathcal{G}(\alpha\beta\rho; \lambda\tau\sigma; \tau_1 - \tau_2) f(\lambda\tau\sigma; \gamma\delta) \quad (216)$$

where some indices have been changed for convenience,

$$\begin{aligned} \Lambda(\gamma\beta\rho; \tau_2 - \tau_1) &= \left\langle T[\widehat{b}_\gamma(\tau_2) \widehat{b}_\beta^+(\tau_1) \widehat{a}_\rho(\tau_1)] \right\rangle_\beta \\ \Lambda(\alpha\delta\rho; \tau_1 - \tau_2) &= \left\langle T[\widehat{b}_\alpha(\tau_1) \widehat{b}_\delta^+(\tau_2) \widehat{a}_\rho(\tau_1)] \right\rangle_\beta \end{aligned} \quad (217)$$

which are given by Eqs. (210) and (211) with setting $j_\lambda = 0$ and

$$\begin{aligned} \mathcal{G}(\lambda\tau\rho; \gamma\delta\sigma; \tau_1 - \tau_2) &= \frac{\delta^2}{\delta j_\rho^*(\tau_1) \delta j_\sigma^*(\tau_2)} \mathcal{G}(\lambda\tau; \gamma\delta; \tau_1 - \tau_2)^{j\lambda} \big|_{j_\lambda=0} \\ &= \left\langle T\{N[\widehat{b}_\lambda(\tau_1) \widehat{b}_\tau^+(\tau_1) \widehat{a}_\rho(\tau_1)] N[\widehat{b}_\gamma(\tau_2) \widehat{b}_\delta^+(\tau_2) \widehat{a}_\sigma(\tau_2)]\} \right\rangle_\beta \end{aligned} \quad (218)$$

is the six-point Green function including two gluon operators in it. According to the definition in Eq. (180), we have

$$\mathcal{G}(\lambda\tau\rho; \gamma\delta\sigma; \tau_1 - \tau_2) = G(\lambda\tau\rho; \gamma\delta\sigma; \tau_1 - \tau_2) - \Lambda(\lambda\tau\rho) \Lambda(\gamma\delta\sigma) \quad (219)$$

where

$$G(\lambda\tau\rho; \gamma\delta\sigma; \tau_1 - \tau_2) = \langle T[\widehat{b}_\lambda(\tau_1) \widehat{b}_\tau^+(\tau_1) \widehat{a}_\rho(\tau_1) \widehat{b}_\gamma(\tau_2) \widehat{b}_\delta^+(\tau_2) \widehat{a}_\sigma(\tau_2)] \rangle_\beta \quad (220)$$

is the ordinary six-point Green function. It should be noted that due to the restriction of the delta function, the terms in the brackets on the right hand sides of Eqs. (215) and (216) actually are "time"-independent.

It is easy to see that the Green functions $\mathcal{G}(\alpha\beta; \lambda\tau\sigma; \tau_1 - \tau_2)$ and $\mathcal{G}(\lambda\tau\rho; \gamma\delta\sigma; \tau_1 - \tau_2)$, as the Green functions $\mathcal{G}(\alpha\beta; \gamma\delta; \tau_1 - \tau_2)$ and $\mathcal{G}(\alpha\beta\rho; \gamma\delta; \tau_1 - \tau_2)$, are periodic. Therefore, by the Fourier transformation, i.e. by the integration $\int_0^\beta d\tau e^{i\omega_n \tau}$, noticing $d/d\tau_2 = -d/d\tau$, Eqs. (215) and (216) will be transformed to

$$(i\omega_n + \theta_\gamma \varepsilon_\gamma - \theta_\delta \varepsilon_\delta) \mathcal{G}(\alpha\beta; \gamma\delta; \omega_n) = S(\alpha\beta; \gamma\delta) + \sum_{\lambda\tau\sigma} \mathcal{G}(\alpha\beta; \lambda\tau\sigma; \omega_n) f(\lambda\tau\sigma; \gamma\delta) \quad (221)$$

where $S(\alpha\beta; \gamma\delta)$ was defined in Eq. (179) and

$$(i\omega_n + \theta_\gamma \varepsilon_\gamma - \theta_\delta \varepsilon_\delta) \mathcal{G}(\alpha\beta\rho; \gamma\delta; \omega_n) = R(\alpha\beta\rho; \gamma\delta) + \sum_{\lambda\tau\sigma} \mathcal{G}(\alpha\beta\rho; \lambda\tau\sigma; \omega_n) f(\lambda\tau\sigma; \gamma\delta) \quad (222)$$

where

$$R(\alpha\beta\rho; \gamma\delta) = \delta_{\alpha\delta} \Lambda(\gamma\beta\rho) - \delta_{\beta\gamma} \Lambda(\alpha\delta\rho) = \langle [\hat{b}_\alpha \hat{b}_\beta^+, \hat{b}_\gamma \hat{b}_\delta^+] - \hat{a}_\rho \rangle_\beta \quad (223)$$

which is "time"-independent.

Now we are ready to derive the interaction kernel. Acting on the both sides of Eq. (186) with $(i\omega_n + \theta_\gamma \varepsilon_\gamma - \theta_\delta \varepsilon_\delta)$ and using Eqs. (221) and (222), one gets

$$\sum_{\mu\nu} K(\alpha\beta; \mu\nu; \omega_n) S(\mu\nu; \gamma\delta) = \sum_{\lambda\tau\rho} f(\alpha\beta; \lambda\tau\rho) R(\lambda\tau\rho; \gamma\delta) + \sum_{\lambda\tau\rho} \sum_{\xi\eta\sigma} f(\alpha\beta; \lambda\tau\rho) \times \mathcal{G}(\lambda\tau\rho; \xi\eta\sigma; \omega_n) f(\xi\eta\sigma; \gamma\delta) - \sum_{\mu\nu} \sum_{\xi\eta\sigma} K(\alpha\beta; \mu\nu; \omega_n) \mathcal{G}(\alpha\beta; \xi\eta\sigma; \omega_n) f(\lambda\tau\sigma; \gamma\delta). \quad (224)$$

Operating on the both sides of Eq. (186) with the inverse of $\mathcal{G}(\mu\nu; \gamma\delta; \omega_n)$, we have

$$K(\alpha\beta; \gamma\delta; \omega_n) = \sum_{\gamma\delta} \sum_{\lambda\tau\sigma} f(\alpha\beta; \lambda\tau\rho) \mathcal{G}(\lambda\tau\rho; \mu\nu; \omega_n) \mathcal{G}^{-1}(\mu\nu; \gamma\delta; \omega_n). \quad (225)$$

Upon substituting Eq. (225) onto the right hand side of Eq. (224) and acting on Eq. (224) with the inverse $S^{-1}(\mu\nu; \gamma\delta)$, we eventually arrive at

$$K(\alpha\beta; \gamma\delta; E) = \sum_{\mu\nu} \left\{ \sum_{\lambda\tau\rho} f(\alpha\beta; \lambda\tau\rho) R(\lambda\tau\rho; \mu\nu) + \sum_{\lambda\tau\rho} \sum_{\xi\eta\sigma} f(\alpha\beta; \lambda\tau\rho) \mathcal{G}(\lambda\tau\rho; \xi\eta\sigma; E) f(\xi\eta\sigma; \mu\nu) - \sum_{\lambda\tau\rho} \sum_{\xi\eta\sigma} \sum_{\kappa\varsigma} \sum_{\pi\theta} f(\alpha\beta; \lambda\tau\rho) \mathcal{G}(\lambda\tau\rho; \kappa\varsigma; E) \mathcal{G}^{-1}(\kappa\varsigma; \pi\theta; E) \mathcal{G}(\pi\theta; \xi\eta\sigma; E) f(\xi\eta\sigma; \mu\nu) \right\} S^{-1}(\mu\nu; \gamma\delta) \quad (226)$$

where ω_n has been replaced by E . This just is the wanted closed expression of the interaction kernel appearing in Eq. (192). In accordance with Eq. (186), the last term in Eq. (226) can be written in the form

$$\sum_{\rho\sigma} \sum_{\xi\eta} \sum_{\mu\nu} K(\alpha\beta; \rho\sigma; E) \mathcal{G}(\rho\sigma; \xi\eta; E) K(\xi\eta; \mu\nu; E) S^{-1}(\mu\nu; \gamma\delta) \quad (227)$$

which exhibits a typical B-S reducible structure [17]. Therefore, the last term in Eq. (226) plays the role of cancelling the B-S reducible part contained in the other terms in Eq. (226) to make the kernel to be B-S irreducible. If we use the above expression in place of the last term in Eq. (226) and acting on Eq. (226) with $S(\gamma\delta; \mu\nu)$, we obtain from Eq. (226) an integral equation satisfied by the kernel $K(\alpha\beta; \gamma\delta; E)$. Define

$$\mathcal{R}(\alpha\beta; \gamma\delta) = \sum_{\lambda\tau\rho} f(\alpha\beta; \lambda\tau\rho) R(\lambda\tau\rho; \gamma\delta) \quad (228)$$

and

$$\mathcal{Q}(\alpha\beta; \gamma\delta) = \sum_{\lambda\tau\rho} \sum_{\xi\eta\sigma} f(\alpha\beta; \lambda\tau\rho) \mathcal{G}(\lambda\tau\rho; \xi\eta\sigma; E) f(\xi\eta\sigma; \gamma\delta), \quad (229)$$

the integral equation can be written in the matrix form as follows

$$KS = \mathcal{R} + \mathcal{Q} - K\mathcal{G}K. \quad (230)$$

For comparison with the kernel in Eq. (226) and for convenience of nonperturbative investigations, we would like to show the corresponding closed expression given in the position space without giving derivation. This kernel can be obtained from the kernel in Eq. (226) by making use of the inverse of the Fourier transformations written in sect. 3

or derived from the generating functional represented in the position space (see Appendix) by completely following the procedure as described in this section. The kernel is represented as follows:

$$K(\vec{x}_1, \vec{x}_2; \vec{y}_1, \vec{y}_2; E) = \int d^3 z_1 d^3 z_2 \{ \mathcal{R}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2) + \mathcal{Q}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E) - \mathcal{D}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E) \} S^{-1}(\vec{z}_1, \vec{z}_2; \vec{y}_1, \vec{y}_2) \quad (231)$$

where $\mathcal{R}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2)$, $\mathcal{Q}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E)$ and $\mathcal{D}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E)$ are separately described below.

The function $\mathcal{R}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2)$ can be represented as

$$\mathcal{R}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2) = \sum_{i=1}^2 \Omega_i^{a\mu} \mathcal{R}_\mu^{(i)a}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2) \quad (232)$$

in which

$$\Omega_1^{a\mu} = ig\gamma_1^4 \gamma_1^\mu T_1^a, \quad \Omega_2^{b\nu} = ig\gamma_2^4 \gamma_2^\nu \bar{T}_2^b \quad (233)$$

with $T_1^a = \lambda^a/2$ and $\bar{T}_2^b = -\lambda^{a*}/2$ being the quark and antiquark color matrices respectively and

$$\mathcal{R}_\mu^{(i)a}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2) = \delta^3(\vec{x}_1 - \vec{z}_1) \gamma_1^4 \Lambda_\mu^{ca}(\vec{x}_i | \vec{x}_2, \vec{z}_2) + \delta^3(\vec{x}_2 - \vec{z}_2) \gamma_2^4 \Lambda_\mu^a(\vec{x}_i | \vec{x}_1, \vec{z}_1) \quad (234)$$

here $\Lambda_\mu^a(\vec{x}_i | \vec{x}_1, \vec{y}_1)$ and $\Lambda_\mu^{ca}(\vec{x}_i | \vec{x}_2, \vec{y}_2)$ are defined as

$$\begin{aligned} \Lambda_\mu^a(\vec{x}_i | \vec{x}_1, \vec{y}_1) &= \langle T[\mathbf{A}_\mu^a(\vec{x}_i, \tau_1) \psi(\vec{x}_1, \tau_1) \bar{\psi}(\vec{y}_1, \tau_1)] \rangle_\beta, \\ \Lambda_\mu^{ca}(\vec{x}_i | \vec{x}_2, \vec{y}_2) &= \langle T[\mathbf{A}_\mu^a(\vec{x}_i, \tau_1) \psi^c(\vec{x}_2, \tau_1) \bar{\psi}^c(\vec{y}_2, \tau_1)] \rangle_\beta \end{aligned} \quad (235)$$

which are time-independent due to the translation-invariance property of the Green functions.

The function is of the form

$$\mathcal{Q}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E) = \sum_{i,j=1}^2 \Omega_i^{a\mu} \mathcal{G}_{\mu\nu}^{ab}(\vec{x}_i, \vec{z}_j | \vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E) \bar{\Omega}_j^{b\nu} \quad (236)$$

in which

$$\bar{\Omega}_1^{a\mu} = ig\gamma_1^\mu \gamma_1^4 T_1^a, \quad \bar{\Omega}_2^{a\mu} = ig\gamma_2^\mu \gamma_2^4 \bar{T}_2^a, \quad (237)$$

$\mathcal{G}_{\mu\nu}^{ab}(\vec{x}_i, \vec{z}_j | \vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E)$ is the Fourier transform of the Green function defined by

$$\begin{aligned} \mathcal{G}_{\mu\nu}^{ab}(\vec{x}_i, \vec{z}_j | \vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; \tau_1 - \tau_2) \\ = \langle T\{N[\mathbf{A}_\mu^a(\vec{x}_i, \tau_1) \psi(\vec{x}_1, \tau_1) \psi^c(\vec{x}_2, \tau_1)] N[\mathbf{A}_\nu^b(\vec{z}_j, \tau_2) \bar{\psi}(\vec{z}_1, \tau_2) \bar{\psi}^c(\vec{z}_2, \tau_2)]\} \rangle_\beta \end{aligned} \quad (238)$$

The function $\mathcal{D}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E)$ is expressed by

$$\begin{aligned} \mathcal{D}(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2; E) &= \int \prod_{k=1}^2 d^3 u_k d^3 v_k \sum_{i,j=1}^2 \Omega_i^{a\mu} \mathcal{G}_\mu^{(i)a}(\vec{x}_i | \vec{x}_1, \vec{x}_2; \vec{u}_1, \vec{u}_2; E) \\ &\times \mathcal{G}^{-1}(\vec{u}_1, \vec{u}_2; \vec{v}_1, \vec{v}_2; E) \mathcal{G}_\nu^{(j)b}(\vec{z}_j | \vec{v}_1, \vec{v}_2; \vec{z}_1, \vec{z}_2; E) \bar{\Omega}_j^{b\nu} \end{aligned} \quad (239)$$

in which $\mathcal{G}_\mu^{(i)a}(\vec{x}_i | \vec{x}_1, \vec{x}_2; \vec{u}_1, \vec{u}_2; E)$ and $\mathcal{G}_\nu^{(j)b}(\vec{z}_j | \vec{v}_1, \vec{v}_2; \vec{z}_1, \vec{z}_2; E)$ are the Fourier transforms of the following Green functions

$$\begin{aligned} \mathcal{G}_\mu^{(i)a}(\vec{x}_i | \vec{x}_1, \vec{x}_2; \vec{u}_1, \vec{u}_2; \tau_1 - \tau_2) \\ = \langle T\{N[\mathbf{A}_\mu^a(\vec{x}_i, \tau_1) \psi(\vec{x}_1, \tau_1) \psi^c(\vec{x}_2, \tau_1)] N[\bar{\psi}(\vec{u}_1, \tau_2) \bar{\psi}^c(\vec{u}_2, \tau_2)]\} \rangle_\beta \end{aligned} \quad (240)$$

and

$$\begin{aligned} \mathcal{G}_\nu^{(j)b}(\vec{z}_j | \vec{v}_1, \vec{v}_2; \vec{z}_1, \vec{z}_2; \tau_1 - \tau_2) \\ = \langle T\{N[\psi(\vec{v}_1, \tau_1) \psi^c(\vec{v}_2, \tau_1)] N[\mathbf{A}_\nu^b(\vec{z}_j, \tau_2) \bar{\psi}(\vec{z}_1, \tau_2) \bar{\psi}^c(\vec{z}_2, \tau_2)]\} \rangle_\beta \end{aligned} \quad (241)$$

The $S^{-1}(\vec{z}_1, \vec{z}_2; \vec{y}_1, \vec{y}_2)$ in Eq. (231) is the inverse of the function defined by

$$S(\vec{x}_1, \vec{x}_2; \vec{z}_1, \vec{z}_2) = \delta^3(\vec{x}_1 - \vec{z}_1) \gamma_1^4 S_F^c(\vec{x}_2 - \vec{z}_2) + \delta^3(\vec{x}_2 - \vec{z}_2) \gamma_2^4 S_F(\vec{x}_1 - \vec{z}_1) \quad (242)$$

in which $S_F(\vec{x}_1 - \vec{z}_1)$ and $S_F^c(\vec{x}_2 - \vec{z}_2)$ are the equal-time quark and antiquark thermal propagators respectively. It is clear that there is one-to-one correspondence between the both kernels written in Eqs. (226) and (231). It is noted that the interaction kernel derived in this section is nonperturbative because the Green functions included in the kernel are defined in the Heisenberg picture. Perturbative calculations of the kernel can easily be done by using the familiar perturbative expansions of Green functions as illustrated in the next section.

VII. ONE GLUON EXCHANGE KERNEL AND HAMILTONIAN

In this section, we would like to show the one-gluon exchange kernel given by the expression in Eq. (226). In the lowest order approximation of perturbation, only the first term of the series in Eq. (194), i.e., the kernel $K(\alpha^+\beta^-; \gamma^+\delta^-; E)$ represented in Eq. (195) is needed to be taken into account in Eqs. (192) and (193). In the lowest order approximation, as seen from Eq. (223), the first term in Eq. (226) vanishes because the function $R(\lambda\tau\rho; \gamma^+\delta^-)$ gives no contribution to the kernel owing to the expectation value $\langle n | \hat{a}_\rho | n \rangle$ vanishes. Therefore, the one-gluon exchange kernel can only arise from the second term in Eq. (226) where the Green function $\mathcal{G}(\lambda\tau\rho; \xi\eta\sigma; E)$ reduces to the ordinary one $G(\lambda\tau\rho; \xi\eta\sigma; E)$ in the lowest order approximation because the last term in Eq. (226) vanishes in this case for the reason as argued for the function $R(\lambda\tau\rho; \gamma^+\delta^-)$. To evaluate the inverse $S^{-1}(\mu\nu; \gamma^+\delta^-)$, we first evaluate $S(\gamma^+\delta^-; \mu\nu)$. From the expression denoted in Eq. (179), it is easy to find that the nonvanishing contribution of $S(\gamma^+\delta^-; \mu\nu)$ is given by

$$\begin{aligned} S(\gamma^+\delta^-; \mu^-\nu^+) &= \delta_{\gamma^+\nu^+} S_{\mu^-\delta^-} - \delta_{\mu^-\delta^-} S_{\gamma^+\nu^+} \\ &= \delta_{\gamma^+\nu^+} \delta_{\mu^-\delta^-} - \delta_{\mu^-\delta^-} S_{\gamma^+\nu^+} - \delta_{\gamma^+\nu^+} \bar{S}_{\delta^-\mu^-} \end{aligned} \quad (243)$$

where $S_{\gamma^+\nu^+}$ and $\bar{S}_{\delta^-\mu^-}$ are the quark and antiquark equal-time propagators. Their expressions, according to the common definition, can be read from Eqs. (139) and (140) by setting $\tau_1 - \tau_2 \rightarrow 0^+$,

$$\begin{aligned} S_{\gamma^+\nu^+} &= \delta_{\gamma^+\nu^+} \Delta_q(\vec{q}_1, 0^+) = \delta_{\gamma^+\nu^+} \frac{1}{2} [\bar{n}_f(\vec{q}_1) - n_f(\vec{q}_1)], \\ \bar{S}_{\delta^-\mu^-} &= \delta_{\delta^-\mu^-} \Delta_q(\vec{q}_2, 0^+) = \delta_{\delta^-\mu^-} \frac{1}{2} [\bar{n}_f(\vec{q}_2) - n_f(\vec{q}_2)]. \end{aligned} \quad (244)$$

Therefore, we can write

$$S^{-1}(\mu^-\nu^+; \gamma^+\delta^-) = \delta_{\gamma^+\nu^+} \delta_{\mu^-\delta^-} S^{-1}(\gamma\delta) \quad (245)$$

where

$$\mathcal{S}(\gamma\delta) = 1 - \frac{1}{2} [\bar{n}_f(\vec{q}_1) - n_f(\vec{q}_1) + \bar{n}_f(\vec{q}_2) - n_f(\vec{q}_2)] \quad (246)$$

Thus, to derive the lowest order approximate kernel, we only need to consider

$$K(\alpha^+\beta^-; \gamma^+\delta^-; E) = \sum_{\lambda\tau\rho} \sum_{\xi\eta\sigma} f(\alpha^+\beta^-; \lambda\tau\rho) G(\lambda\tau\rho; \mu\nu\sigma; E) f(\mu\nu\sigma; \delta^-\gamma^+) S^{-1}(\gamma\delta). \quad (247)$$

From Eqs. (169), (207), (80) and (67), it is clearly seen that when λ, τ, μ and ν take λ^+, τ^-, μ^- and ν^+ , the functions $f(\alpha^+\beta^-; \lambda\tau\rho)$ and $f(\mu\nu\sigma; \delta^-\gamma^+)$ give the quark-antiquark interaction taking place in the t-channel scattering process; while, when λ, τ, μ and ν take λ^-, τ^+, μ^+ and ν^- , the $f(\alpha^+\beta^-; \lambda\tau\rho)$ and $f(\mu\nu\sigma; \delta^-\gamma^+)$ will give the quark-antiquark vertices which describe the $q\bar{q}$ annihilation process. Since the expectation value of the $q\bar{q}$ color matrix appearing in the $q\bar{q}$ lowest order annihilation process is zero in the color singlet, it is only necessary to consider the following interaction kernel

$$\begin{aligned} K(\alpha^+\beta^-; \gamma^+\delta^-; E) &= \sum_{\mu\nu} \sum_{\lambda\tau\rho} \sum_{\xi\eta\sigma} [f(\alpha^+\beta^-; \lambda^+\tau^-\rho^+) G(\lambda^+\tau^-\rho^+; \mu^-\nu^+\sigma^-; E) f(\mu^-\nu^+\sigma^-; \delta^-\gamma^+) \\ &\quad + f(\alpha^+\beta^-; \lambda^+\tau^-\rho^-) G(\lambda^+\tau^-\rho^-; \mu^-\nu^+\sigma^+; E) f(\mu^-\nu^+\sigma^+; \delta^-\gamma^+)] S^{-1}(\gamma\delta). \end{aligned} \quad (248)$$

Noting that the functions $f(\alpha^+\beta^-; \lambda^+\tau^-\rho)$ and $f(\mu^-\nu^+\sigma; \delta^-\gamma^+)$ are proportional to the coupling constant g , in the approximation of order g^2 , we only need to consider the zero-order of the Green function $G(\lambda^+\tau^-\rho^\pm; \mu^-\nu^+\sigma^\mp; E)$. This Green function may easily be derived from the generating functional $Z^0[j]$ represented in Eqs. (96), (119), (138) and (150). The result is

$$\begin{aligned} G(\lambda^+\tau^-\rho^-; \mu^-\nu^+\sigma^+; \tau_1 - \tau_2) &= \frac{1}{Z^0} \frac{\delta^6 Z^0[j]}{\delta\eta_\lambda^*(\tau_1) \delta\bar{\eta}_\tau^*(\tau_1) \delta\bar{\eta}_\mu(\tau_2) \delta\bar{\eta}_\nu(\tau_2) \delta\xi_\rho^*(\tau_1) \delta\xi_\sigma(\tau_2)} \Big|_{j=0} \\ &= \delta_{\lambda\nu} \Delta_q(\vec{k}_\lambda, \tau_1 - \tau_2) \delta_{\tau\mu} \Delta_q(\vec{k}_\tau, \tau_1 - \tau_2) \delta_{\rho\sigma} \Delta_g(\vec{k}_\rho, \tau_1 - \tau_2) \\ &= G(\lambda^+\tau^-\rho^+; \mu^-\nu^+\sigma^-; \tau_1 - \tau_2) \end{aligned} \quad (249)$$

The last equality in the above arises from the fact that $\Delta_g(\vec{k}_\rho, \tau_1 - \tau_2)$ is an even function of $\tau_1 - \tau_2$. With the expressions given in Eqs. (124) and (140), the Fourier transform of $G(\lambda^+\tau^-\rho^-; \mu^-\nu^+\sigma^+; \tau_1 - \tau_2)$ can be found to be

$$G(\lambda^+ \tau^- \rho^-; \mu^- \nu^+ \sigma^+; \omega_n) = \delta_{\lambda\nu} \delta_{\tau\mu} \delta_{\rho\sigma} \frac{1}{8} \Delta(\lambda\tau\rho) \quad (250)$$

where $\Delta(\lambda\tau\rho)$ will be specified soon later.

Substitution of Eqs. (169), (207) and (250) in Eq. (248) leads to

$$\begin{aligned} K(\alpha^+ \beta^-; \gamma^+ \delta^-; E) = & \frac{1}{4} [\sum_{\lambda\rho} A(\alpha^+ \lambda^+ \rho^+) A(\lambda^+ \gamma^+ \rho^-) \Delta(\lambda\beta\rho) \delta_{\beta\delta} \\ & + \sum_{\tau\rho} A(\tau^- \beta^- \rho^+) A(\delta^- \tau^- \rho^-) \Delta(\alpha\tau\rho) \delta_{\alpha\gamma} - \sum_{\rho} A(\alpha^+ \gamma^+ \rho^+) A(\delta^- \beta^- \rho^-) \Delta(\gamma\beta\rho) \\ & - \sum_{\rho} A(\delta^- \beta^- \rho^+) A(\alpha^+ \gamma^+ \rho^-) \Delta(\alpha\delta\rho)] S^{-1}(\gamma\delta). \end{aligned} \quad (251)$$

where the first two terms on the right hand side are unconnected and represent the quark and antiquark self-energies, while the remaining two terms precisely give the t-channel one-gluon exchange kernel. In view of the expression in Eq. (80) and the definitions in Eqs. (67) and (71), we can write

$$A(\alpha^+ \gamma^+ \rho^\pm) = ig(2\pi)^3 \delta^3(\vec{p}_1 - \vec{q}_1 \mp \vec{k}) \bar{u}_{\sigma_1}(\vec{p}_1) T^c \gamma_\mu u_{\sigma'_1}(\vec{q}_1) (2\pi)^{-3/2} (2\omega(\vec{k}))^{-1/2} \varepsilon_\mu^\lambda(\vec{k}) \quad (252)$$

and

$$\begin{aligned} A(\delta^- \beta^- \rho^\mp) = & ig(2\pi)^3 \delta^3(\vec{q}_2 - \vec{p}_2 \pm \vec{k}) \bar{v}_{\sigma'_2}(\vec{q}_2) T^c \gamma_\mu v_{\sigma_2}(\vec{p}_2) (2\pi)^{-3/2} (2\omega(\vec{k}))^{-1/2} \varepsilon_\mu^\lambda(\vec{k}) \\ = & -ig(2\pi)^3 \delta^3(\vec{q}_2 - \vec{p}_2 \pm \vec{k}) \bar{u}_{\sigma_2}(\vec{p}_2) (-T^{c*}) \gamma_\mu u_{\sigma'_2}(\vec{q}_2) (2\pi)^{-3/2} (2\omega(\vec{k}))^{-1/2} \varepsilon_\mu^{\lambda'}(\vec{k}) \end{aligned} \quad (253)$$

where the last equality in Eq. (253) is obtained by the charge conjugation transformation. In the above, we have set $\vec{k}_\alpha = \vec{p}_1$, $\vec{k}_\beta = \vec{p}_2$, $\vec{k}_\gamma = \vec{q}_1$, $\vec{k}_\delta = \vec{q}_2$ and $\vec{k}_\rho = \vec{k}$. From Eqs. (249), (250), (124) and (140), it is easy to get

$$\begin{aligned} \Delta(\alpha\delta\rho) = & \frac{1}{2} \int_{-\beta}^{\beta} d\tau e^{i\omega_n \tau} \Delta_q(\vec{p}_1, \tau) \Delta_q(\vec{q}_2, \tau) \Delta_g(\vec{k}, \tau) \\ = & \bar{n}_f^\alpha \bar{n}_f^\delta \bar{n}_b^\rho \frac{e^{-\beta(\varepsilon_\alpha + \varepsilon_\delta + \omega_\rho)} - 1}{i\omega_n - \varepsilon_\alpha - \varepsilon_\delta - \omega_\rho} - \bar{n}_f^\alpha \bar{n}_f^\delta n_b^\rho \frac{e^{-\beta(\varepsilon_\alpha + \varepsilon_\delta - \omega_\rho)} - 1}{i\omega_n - \varepsilon_\alpha - \varepsilon_\delta + \omega_\rho} \\ & - \bar{n}_f^\alpha n_f^\delta \bar{n}_b^\rho \frac{e^{-\beta(\varepsilon_\alpha - \varepsilon_\delta + \omega_\rho)} - 1}{i\omega_n - \varepsilon_\alpha + \varepsilon_\delta - \omega_\rho} + \bar{n}_f^\alpha n_f^\delta n_b^\rho \frac{e^{-\beta(\varepsilon_\alpha - \varepsilon_\delta - \omega_\rho)} - 1}{i\omega_n - \varepsilon_\alpha + \varepsilon_\delta + \omega_\rho} \\ & - n_f^\alpha \bar{n}_f^\delta \bar{n}_b^\rho \frac{e^{\beta(\varepsilon_\alpha - \varepsilon_\delta - \omega_\rho)} - 1}{i\omega_n + \varepsilon_\alpha - \varepsilon_\delta - \omega_\rho} + n_f^\alpha \bar{n}_f^\delta n_b^\rho \frac{e^{\beta(\varepsilon_\alpha - \varepsilon_\delta + \omega_\rho)} - 1}{i\omega_n + \varepsilon_\alpha - \varepsilon_\delta + \omega_\rho} \\ & + n_f^\alpha n_f^\delta \bar{n}_b^\rho \frac{e^{\beta(\varepsilon_\alpha + \varepsilon_\delta - \omega_\rho)} - 1}{i\omega_n + \varepsilon_\alpha + \varepsilon_\delta - \omega_\rho} - n_f^\alpha n_f^\delta n_b^\rho \frac{e^{\beta(\varepsilon_\alpha + \varepsilon_\delta + \omega_\rho)} - 1}{i\omega_n + \varepsilon_\alpha + \varepsilon_\delta + \omega_\rho} \\ \equiv & \Delta(\vec{p}_1, \vec{q}_2, \vec{k}; \omega_n) \end{aligned} \quad (254)$$

and

$$\begin{aligned} \Delta(\gamma\beta\rho) = & \frac{1}{2} \int_{-\beta}^{\beta} d\tau e^{i\omega_n \tau} \Delta_q(\vec{p}_2, \tau) \Delta_q(\vec{q}_1, \tau) \Delta_g(\vec{k}, \tau) \\ = & \bar{n}_f^\gamma \bar{n}_f^\beta \bar{n}_b^\rho \frac{e^{-\beta(\varepsilon_\gamma + \varepsilon_\beta + \omega_\rho)} - 1}{i\omega_n - \varepsilon_\gamma - \varepsilon_\beta - \omega_\rho} - \bar{n}_f^\gamma \bar{n}_f^\beta n_b^\rho \frac{e^{-\beta(\varepsilon_\gamma + \varepsilon_\beta - \omega_\rho)} - 1}{i\omega_n - \varepsilon_\gamma - \varepsilon_\beta + \omega_\rho} \\ & - \bar{n}_f^\gamma n_f^\beta \bar{n}_b^\rho \frac{e^{-\beta(\varepsilon_\gamma - \varepsilon_\beta + \omega_\rho)} - 1}{i\omega_n - \varepsilon_\gamma + \varepsilon_\beta - \omega_\rho} + \bar{n}_f^\gamma n_f^\beta n_b^\rho \frac{e^{-\beta(\varepsilon_\gamma - \varepsilon_\beta - \omega_\rho)} - 1}{i\omega_n - \varepsilon_\gamma + \varepsilon_\beta + \omega_\rho} \\ & - n_f^\gamma \bar{n}_f^\beta \bar{n}_b^\rho \frac{e^{\beta(\varepsilon_\gamma - \varepsilon_\beta - \omega_\rho)} - 1}{i\omega_n + \varepsilon_\gamma - \varepsilon_\beta - \omega_\rho} + n_f^\gamma \bar{n}_f^\beta n_b^\rho \frac{e^{\beta(\varepsilon_\gamma - \varepsilon_\beta + \omega_\rho)} - 1}{i\omega_n + \varepsilon_\gamma - \varepsilon_\beta + \omega_\rho} \\ & + n_f^\gamma n_f^\beta \bar{n}_b^\rho \frac{e^{\beta(\varepsilon_\gamma + \varepsilon_\beta - \omega_\rho)} - 1}{i\omega_n + \varepsilon_\gamma + \varepsilon_\beta - \omega_\rho} - n_f^\gamma n_f^\beta n_b^\rho \frac{e^{\beta(\varepsilon_\gamma + \varepsilon_\beta + \omega_\rho)} - 1}{i\omega_n + \varepsilon_\gamma + \varepsilon_\beta + \omega_\rho} \\ \equiv & \Delta(\vec{p}_2, \vec{q}_1, \vec{k}; \omega_n) \end{aligned} \quad (255)$$

where

$$\varepsilon_\alpha = \sqrt{\vec{p}_1^2 + m_1}, \quad \varepsilon_\beta = \sqrt{\vec{p}_2^2 + m_2}, \quad \varepsilon_\gamma = \sqrt{\vec{q}_1^2 + m_1}, \quad \varepsilon_\delta = \sqrt{\vec{q}_2^2 + m_2}, \quad \omega_\rho = |\vec{k}| \quad (256)$$

It is noted here that the chemical potential is not taken into account for the bound state. Another expressions of the functions $\Delta(\alpha\delta\rho)$ and $\Delta(\gamma\beta\rho)$ may be given by making use of the expansions presented in Eqs. (A10) and (A19) in the Appendix. Since the expressions contain infinite series, it might be not convenient for our purpose. Upon inserting Eqs. (252)-(255) into the last two terms in Eq. (251) and noticing the definition in Eq. (72), after completing the integration over \vec{k} and the summation over the polarization index, we obtain

$$\begin{aligned} K(\alpha^+ \beta^-; \gamma^+ \delta^-; E) = & K(\vec{p}_1, \vec{p}_2; \vec{q}_1, \vec{q}_2; E) \\ = & (2\pi)^{-3} \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{q}_1 - \vec{q}_2) V(\vec{p}_1, \vec{p}_2; \vec{q}_1, \vec{q}_2; E) \end{aligned} \quad (257)$$

where the $i\omega_n$ in Eqs. (254) and (255) has been replaced by E and

$$V(\vec{p}_1, \vec{p}_2; \vec{q}_1, \vec{q}_2; E) = \bar{u}_{\sigma_1}(\vec{p}_1) T^c \gamma_\mu u_{\sigma'_1}(\vec{q}_1) \bar{u}_{\sigma_2}(\vec{p}_2) T^{c*} \gamma_\mu u_{\sigma'_2}(\vec{q}_2) D(\vec{p}_1, \vec{p}_2; \vec{q}_1, \vec{q}_2) \quad (258)$$

in which

$$D(\vec{p}_1, \vec{p}_2; \vec{q}_1, \vec{q}_2) = \frac{1}{8\omega(\vec{p}_1 - \vec{q}_1)} [\Delta(\vec{p}_1, \vec{q}_2, \vec{p}_1 - \vec{q}_1) + \Delta(\vec{p}_2, \vec{q}_1, \vec{p}_1 - \vec{q}_1)] \mathcal{S}^{-1}(\vec{q}_1, \vec{q}_2) \quad (259)$$

With the kernel given above, the equation in Eq. (192) can be written as

$$[E - \varepsilon(\vec{p}_1) - \varepsilon(\vec{p}_2)] \chi_{P\alpha}(\vec{p}_1, \vec{p}_2) = \int d^3 q_1 d^3 q_2 K(\vec{p}_1, \vec{p}_2; \vec{q}_1, \vec{q}_2) \chi_{P\alpha}(\vec{q}_1, \vec{q}_2) \quad (260)$$

When we introduce the cluster momenta

$$\begin{aligned} \vec{P} &= \vec{p}_1 + \vec{p}_2, \quad \vec{Q} = \vec{q}_1 + \vec{q}_2, \quad \vec{q} = \eta_2 \vec{p}_1 - \eta_1 \vec{p}_2, \\ \vec{k} &= \eta_2 \vec{q}_1 - \eta_1 \vec{q}_2, \quad \eta_1 = \frac{m_1}{m_1 + m_2}, \quad \eta_2 = \frac{m_2}{m_1 + m_2}, \end{aligned} \quad (261)$$

in the center of mass frame, Eq. (260) will be represented as

$$[E - \varepsilon(\vec{p}_1) - \varepsilon(\vec{p}_2)] \chi_{P\alpha}(\vec{q}) = \int \frac{d^3 k}{(2\pi)^3} V(\vec{P}, \vec{q}, \vec{k}) \chi_{P\alpha}(\vec{k}). \quad (262)$$

As we know, the spinor function in Eq. (258) can be written as $u_\sigma(\vec{p}) = U(\vec{p}) \varphi_\sigma$ where φ_σ is the spin wave function and $U(\vec{p})$ is the ordinary Dirac spinor determined by the Dirac equation. The spin wave functions may be absorbed into the amplitude $\psi(\alpha\beta; E)$ appearing in Eq. (193). Thus, corresponding to the equation in Eq. (262), the equation in Eq. (193) will be represented as

$$[E - \varepsilon_1(\vec{q}) - \varepsilon_2(\vec{q})] \psi_{P\alpha}(\vec{q}) = \int \frac{d^3 k}{(2\pi)^3} \hat{V}(\vec{P}; \vec{q}, \vec{k}) \psi_{P\alpha}(\vec{k}) \quad (263)$$

where $\psi_{P\alpha}(\vec{q})$ stands for the color singlet wave function given in the Pauli spinor space and

$$\hat{V}(\vec{P}; \vec{q}, \vec{k}) = -\frac{4}{3} g^2 \bar{U}(\vec{p}_1) \gamma_\mu U(\vec{q}_1) \bar{U}(\vec{p}_2) \gamma^\mu U(\vec{q}_2) D(\vec{P}, \vec{q}, \vec{k}) \quad (264)$$

represents the one-gluon exchange interaction Hamiltonian which formally is the same as given in the case of zero-temperature [17]. In Eq. (264), we have recovered the Minkowski metric for the γ -matrix in order to compare with the ordinary zero-temperature result and considered that the expectation value of the $q\bar{q}$ color operator $T_1^a(-T_2^{a*})$ in the color singlet equals to $-\frac{4}{3}$.

VIII. CONCLUDING REMARKS

In this paper, there are two new achievements. One is that the path-integral formalism of the thermal QCD has been correctly established in the coherent-state representation. The expression of the QCD generating functional formulated in this paper not only gives an alternative quantization of the QCD, but also provides a general method of calculating the partition function, the thermal Green functions and thereby other statistical quantities of QCD in the coherent-state representation. In particular, the generating functional enables us to carry out analytical calculations without being concerned with its discretized form. As one has seen from Sect. 4, the analytical calculation of the zero-order generating functional is more simple and direct than the previous calculations performed in the discretized form given either in the coherent-state representation or in the position space [18-22]. The coherent-state path-integral formalism corresponds to the operator formalism represented in terms of creation and annihilation operators. In comparison with the latter formalism which was frequently applied in the many-body theory [21, 29], the coherent-state path-integral formalism has a prominent advantage that in calculations within this formalism, use of the operator commutators and the Wick theorem is completely avoided. Therefore, it is more convenient for practical applications. It should be noted that although the QCD generating functional is derived in the Feynman gauge, the result is exact. This is because QCD is a gauge-independent theory. As shown in Sect. 4, in the partition function derived in the Feynman gauge, the unphysical part of the partition function given by the unphysical degrees of freedom of gluons is completely cancelled out by the partition function arising from the ghost particles. Certainly, the generating functional formulated in the coherent-state representation can be established in arbitrary gauges. But, in this case,

the gluon propagator would have a rather complicated form due to that the longitudinal part of the propagator will involve the polarization vector. Another point we would like to mention is that to formulate the quantization of the thermal QCD in the coherent-state representation, we limit ourself to work in the imaginary-time formalism. It is no doubt that the theory can also be described in the real-time formalism. We leave the discussion on this subject in the future.

The main achievement of this paper is the foundation of a rigorous three-dimensional equation for the $q\bar{q}$ bound states at finite temperature. Especially, the interaction kernel in the equation is given a closed expression in the coherent-state representation. This kernel, as shown in Eq. (226), contains only a few types of Green functions with some definite coefficients. We also give the corresponding closed expression represented in the position space. As shown in Eqs. (231)-(242), in this expression only a few types of Green functions and commutators are involved. Therefore, the kernel can not only easily be calculated by the perturbation method, but also is suitable for nonperturbative investigations by using the lattice gauge approach and some others. Since the kernel contains all the interactions taking place in the bound state, obviously, the kernel and the equation presented in the preceding sections are much suitable to study the quark deconfinement at high temperature which is nowadays an important theoretical problem in the high energy physics. It is expected that an accurate nonperturbative calculation of the kernel could come up in the future so as to give the problem of quark deconfinement a definitet solution

IX. ACKNOWLEDGMENT

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X. APPENDIX: DERIVATION OF THE GENERATING FUNCTIONAL REPRESENTED IN THE POSITION SPACE

To confirm the correctness of the results derived in Sect.4, in this appendix, we plan to derive the familiar perturbative expansion of the thermal QCD generating functional represented in the position space from the corresponding one given in section 4. For this purpose, we need to derive the generating functional represented in position space for the free system which can be written as

$$Z^0[J] = Z_g^0[J_\mu^a] Z_q^0[I, \bar{I}] Z_c^0[K^a, \bar{K}^a] \quad (A1)$$

where $Z_g^0[J_\mu^a]$, $Z_q^0[I, \bar{I}]$ and $Z_c^0[K^a, \bar{K}^a]$ are the position space generating functionals arising respectively from the free gluons, quarks and ghost particles and J_μ^a , I , \bar{I} , K^a and \bar{K}^a are the sources coupled to gluon, quark and ghost particle fields respectively. In order to write out the $Z_q^0[I, \bar{I}]$, $Z_g^0[J_\mu^a]$ and $Z_c^0[K^a, \bar{K}^a]$ from the generating functionals given in Eqs. (119), (138) and (150), it is necessary to establish relations between the sources introduced in the position space and in the coherent-state representation. Let us separately discuss the functionals $Z_q^0[I, \bar{I}]$, $Z_g^0[J_\mu^a]$ and $Z_c^0[K^a, \bar{K}^a]$. First we focus our attention on the functional $Z_q^0[I, \bar{I}]$. Usually, the external source terms of fermions in the generating functional given in the position space are of the form $\int_0^\beta d\tau \int d^3x [\bar{I}(\vec{x}, \tau) \psi(\vec{x}, \tau) + \bar{\psi}(\vec{x}, \tau) I(\vec{x}, \tau)]$ [22, 32]. Substituting in this expression the Fourier expansions in Eqs. (58) and (59) for the quark fields and in the following for the sources

$$\begin{aligned} I(\vec{x}, \tau) &= \int \frac{d^3k}{(2\pi)^{3/2}} I(\vec{k}, \tau) e^{i\vec{k} \cdot \vec{x}} \\ \bar{I}(\vec{x}, \tau) &= \int \frac{d^3k}{(2\pi)^{3/2}} \bar{I}(\vec{k}, \tau) e^{-i\vec{k} \cdot \vec{x}} \end{aligned} \quad (A2)$$

we have

$$\begin{aligned} \int_0^\beta d\tau \int d^3x [\bar{I}(\vec{x}, \tau) \psi(\vec{x}, \tau) + \bar{\psi}(\vec{x}, \tau) I(\vec{x}, \tau)] &= \int_0^\beta d\tau \int d^3k [\eta_s^*(\vec{k}, \tau) b_s(\vec{k}, \tau) \\ &+ b_s^*(\vec{k}, \tau) \eta_s(\vec{k}, \tau) + \bar{\eta}_s^*(\vec{k}, \tau) d_s(\vec{k}, \tau) + d_s^*(\vec{k}, \tau) \bar{\eta}_s(\vec{k}, \tau)] \end{aligned} \quad (A3)$$

where

$$\begin{aligned} \eta_s(\vec{k}, \tau) &= \bar{u}_s(\vec{k}) I(\vec{k}, \tau), \quad \eta_s^*(\vec{k}, \tau) = \bar{I}(\vec{k}, \tau) u_s(\vec{k}), \\ \bar{\eta}_s(\vec{k}, \tau) &= -\bar{I}(-\vec{k}, \tau) v_s(\vec{k}), \quad \bar{\eta}_s^*(\vec{k}, \tau) = -\bar{v}_s(\vec{k}) I(-\vec{k}, \tau). \end{aligned} \quad (A4)$$

From these relations and the property of Dirac spinors, the relation in Eq. (137) is easily proved. On substituting Eq. (A4) into Eq. (138), one can get

$$Z_q^0[I, \bar{I}] = Z_q^0 \exp\left\{\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int d^3 k \bar{I}(\vec{k}, \tau_1) S_F(\vec{k}, \tau_1 - \tau_2) I(\vec{k}, \tau_2)\right\} \quad (\text{A5})$$

where

$$S_F(\vec{k}, \tau_1 - \tau_2) = [(\mathbf{k} + m)/\varepsilon(\vec{k})] \Delta_q(\vec{k}, \tau_1 - \tau_2) \quad (\text{A6})$$

with $\mathbf{k} = \gamma_\mu k^\mu$ and $k^\mu = (\vec{k}, i\varepsilon_n)$. By making use of the inverse transformation of Eq. (A2), the generating functional in Eq. (A5) is finally represented as [22, 32]

$$Z_q^0[I, \bar{I}] = Z_q^0 \exp\left\{\int_0^\beta d^4 x_1 \int_0^\beta d^4 x_2 \bar{I}(x_1) S_F(x_1 - x_2) I(x_2)\right\} \quad (\text{A7})$$

where $x = (\vec{x}, \tau)$, $d^4 x = d\tau d^3 x$ and

$$S_F(x_1 - x_2) = \int \frac{d^3 k}{(2\pi)^3} S_F(\vec{k}, \tau_1 - \tau_2) e^{i\vec{k} \cdot \vec{x}} \quad (\text{A8})$$

It is well-known that the propagator $\Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2)$ is antiperiodic,

$$\begin{aligned} \Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2) &= -\Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2 - \beta), \text{ if } \tau_1 \succ \tau_2 \\ \Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2) &= -\Delta_q^{ss'}(\vec{k}, \tau_1 - \tau_2 + \beta), \text{ if } \tau_1 \prec \tau_2 \end{aligned} \quad (\text{A9})$$

This can easily be proved from its representation in the operator formalism as shown in Eq. (46) or Eq. (154) with the help of the translation transformation $\hat{b}_s(\tau) = e^{\beta \hat{K}} \hat{b}_s e^{-\beta \hat{K}}$ and the trace property $Tr(AB) = Tr(BA)$. According to the antiperiodic property of the propagator, we have the following expansion

$$\Delta_q(\vec{k}, \tau) = \frac{1}{\beta} \sum_n \Delta_q(\vec{k}, \varepsilon_n) e^{-i\varepsilon_n \tau} \quad (\text{A10})$$

where $\tau = \tau_1 - \tau_2$, $\varepsilon_n = \frac{\pi}{\beta}(2n + 1)$ with n being the integer and

$$\Delta_q(\vec{k}, \varepsilon_n) = \int_0^\beta d\tau e^{i\varepsilon_n \tau} \Delta_q(\vec{k}, \tau) = \frac{\varepsilon(\vec{k})}{\varepsilon_n^2 + \varepsilon(\vec{k})^2} \quad (\text{A11})$$

Substituting Eq. (A10) into Eq. (A6) and noticing the above expression, it can be found that [22, 32]

$$S_F(x_1 - x_2) = \frac{1}{\beta} \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{e^{i\vec{k} \cdot (\vec{x}_1 - \vec{x}_2) - i\varepsilon_n(\tau_1 - \tau_2)}}{\vec{\gamma} \cdot \vec{k} + m - i\varepsilon_n \gamma^0} \quad (\text{A12})$$

which is the familiar expression of the thermal fermion propagator in the position space.

Next, we discuss the generating functional $Z_g^0[J_\mu^a]$. The source term of gluons in the generating functional given in the position space is commonly taken as $\int_0^\beta d\tau \int d^3 x J_\mu^a(\vec{x}, \tau) A^{a\mu}(\vec{x}, \tau)$ [22, 32]. Employing the expansions in Eq. (60) and in the following

$$J_\mu^a(\vec{x}, \tau) = \int \frac{d^3 k}{(2\pi)^{3/2}} J_\mu^a(\vec{k}, \tau) e^{i\vec{k} \cdot \vec{x}}, \quad (\text{A13})$$

we can write

$$\int_0^\beta d\tau \int d^3 x J_\mu^c(\vec{x}, \tau) A^{c\mu}(\vec{x}, \tau) = \int_0^\beta d\tau \int d^3 k [\xi_\lambda^{c*}(\vec{k}, \tau) a_\lambda^c(\vec{k}, \tau) + a_\lambda^{c*}(\vec{k}, \tau) \xi_\lambda^c(\vec{k}, \tau)] \quad (\text{A14})$$

where

$$\xi_\lambda^c(\vec{k}, \tau) = (2\omega(\vec{k}))^{-1/2} \epsilon_\lambda^\mu(\vec{k}) J_\mu^c(\vec{k}, \tau) = \xi_\lambda^{c*}(-\vec{k}, \tau) \quad (\text{A15})$$

in which the last equality follows from that the $J_\mu^a(\vec{x}, \tau)$ is a real function. Inserting the relations in Eq. (A15) and then the inverse transformation of Eq. (A13) into Eq. (119) and considering completeness of the polarization vectors, one may find the generating functional $Z_g^0[J_\mu^a]$ such that [22, 32]

$$Z_g^0[J_\mu^a] = Z_g^0 \exp\left\{\frac{1}{2} \int_0^\beta d^4x_1 \int_0^\beta d^4x_2 J_\mu^a(x_1) D_{\mu\nu}^{ab}(x_1 - x_2) J_\nu^b(x_2)\right\} \quad (\text{A16})$$

where

$$D_{\mu\nu}^{ab}(x_1 - x_2) = \delta^{ab} \delta_{\mu\nu} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega(\vec{k})} \Delta_g(\vec{k}, \tau_1 - \tau_2) e^{i\vec{k} \cdot (\vec{x}_1 - \vec{x}_2)} \quad (\text{A17})$$

By the same argument as mentioned for Eq. (A9), it can be proved that the gluon propagator $\Delta_g(\vec{k}, \tau_1 - \tau_2)$ is a periodic function

$$\begin{aligned} \Delta_g(\vec{k}, \tau_1 - \tau_2) &= \Delta_g(\vec{k}, \tau_1 - \tau_2 - \beta), \text{ if } \tau_1 \succ \tau_2; \\ \Delta_g(\vec{k}, \tau_1 - \tau_2) &= \Delta_g(\vec{k}, \tau_1 - \tau_2 + \beta), \text{ if } \tau_1 \prec \tau_2. \end{aligned} \quad (\text{A18})$$

Therefore, we have the expansion

$$\Delta_g(\vec{k}, \tau) = \frac{1}{\beta} \sum_n \Delta_g(\vec{k}, \omega_n) e^{-i\omega_n \tau} \quad (\text{A19})$$

where $\omega_n = \frac{2\pi n}{\beta}$ and

$$\Delta_g(\vec{k}, \omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} \Delta_g(\vec{k}, \tau) = \frac{\omega(\vec{k})}{\varepsilon_n^2 + \omega(\vec{k})^2}. \quad (\text{A20})$$

Upon substituting Eqs. (A19) and (A20) in Eq. (A17), we arrive at [22,32]

$$D_{\mu\nu}^{ab}(x_1 - x_2) = \delta^{ab} \frac{1}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{\delta_{\mu\nu}}{\varepsilon_n^2 + \omega(\vec{k})^2} e^{i\vec{k} \cdot (\vec{x}_1 - \vec{x}_2) - i\omega_n(\tau_1 - \tau_2)} \quad (\text{A21})$$

which just is the gluon propagator given in the position space and in the Feynman gauge.

Finally, we turn to the generating functional $Z_c^0[K^a, \overline{K}^a]$. In accordance with the expansions in Eqs. (62) and (63) for the ghost particle fields and those for the external sources:

$$\begin{aligned} K^a(\vec{x}, \tau) &= \int \frac{d^3k}{(2\pi)^{3/2}} K^a(\vec{k}, \tau) e^{i\vec{k} \cdot \vec{x}} \\ \overline{K}^a(\vec{x}, \tau) &= \int \frac{d^3k}{(2\pi)^{3/2}} \overline{K}^a(\vec{k}, \tau) e^{-i\vec{k} \cdot \vec{x}} \end{aligned} \quad (\text{A22})$$

the relation between the sources in the position space and in the coherent-state representation can be found to be

$$\begin{aligned} \int_0^\beta d\tau \int d^3x [\overline{K}^a(\vec{x}, \tau) C^a(\vec{x}, \tau) + \overline{C}^a(\vec{x}, \tau) K^a(\vec{x}, \tau)] &= \int_0^\beta d\tau \int d^3k [\zeta_a^*(\vec{k}, \tau) c_a(\vec{k}, \tau) \\ &+ c_a^*(\vec{k}, \tau) \zeta_a(\vec{k}, \tau) + \overline{\zeta}_a^*(\vec{k}, \tau) \overline{c}_a(\vec{k}, \tau) + \overline{c}_a^*(\vec{k}, \tau) \overline{\zeta}_a(\vec{k}, \tau)] \end{aligned} \quad (\text{A23})$$

where

$$\begin{aligned} \zeta_a(\vec{k}, \tau) &= (2\omega(\vec{k}))^{-1/2} K^a(\vec{k}, \tau), \quad \zeta_a^*(\vec{k}, \tau) = (2\omega(\vec{k}))^{-1/2} \overline{K}^a(\vec{k}, \tau), \\ \overline{\zeta}_a(\vec{k}, \tau) &= -(2\omega(\vec{k}))^{-1/2} \overline{K}^a(-\vec{k}, \tau), \quad \overline{\zeta}_a^*(\vec{k}, \tau) = -(2\omega(\vec{k}))^{-1/2} K^a(-\vec{k}, \tau). \end{aligned} \quad (\text{A24})$$

from which the relations denoted in Eq. (149) directly follows. When the above relations and the inverse transformations of Eq. (A22) are inserted into Eq. (150), one can get

$$Z_c^0[K^a, \overline{K}^a] = Z_c^0 \exp\left\{-\int_0^\beta d^4x_1 \int_0^\beta d^4x_2 \overline{K}^a(x_1) \Delta_c^{ab}(x_1 - x_2) K^b(x_2)\right\} \quad (\text{A25})$$

where

$$\begin{aligned}\Delta_c^{ab}(x_1 - x_2) &= \delta^{ab} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega(\vec{k})} \Delta_g(\vec{k}, \tau_1 - \tau_2) e^{i\vec{k} \cdot (\vec{x}_1 - \vec{x}_2)} \\ &= \delta^{ab} \frac{1}{\beta} \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\varepsilon_n^2 + \omega(\vec{k})^2} e^{i\vec{k} \cdot (\vec{x}_1 - \vec{x}_2) - i\omega_n(\tau_1 - \tau_2)}\end{aligned}\quad (\text{A26})$$

which just is the free ghost particle propagator given in the position space [22, 32]. In the last equality of Eq. (A26), the expansion given in Eqs. (A19) and (A20) have been used.

With the generating functionals given in Eqs. (A7), (A16) and (A25), the zeroth-order generating functional in Eq. (A1) is explicitly represented in terms of the propagators and external sources. Clearly, the exact generating functional can immediately be written out from Eq. (95) as shown in the following

$$Z[J] = \exp\left\{-\int_0^\beta d^4x \mathcal{H}_I\left(\frac{\delta}{\delta J(x)}\right)\right\} Z^0[J] \quad (\text{A27})$$

where J stands for I, \bar{I}, J_μ^a, K^a and \bar{K}^a , $\frac{\delta}{\delta J(x)}$ represents the differentials $\frac{\delta}{\delta I(x)}, -\frac{\delta}{\delta \bar{I}(x)}, \frac{\delta}{\delta J_\mu^a(x)}, \frac{\delta}{\delta K^a(x)}$ and $-\frac{\delta}{\delta \bar{K}^a(x)}$ and $\mathcal{H}_I(\frac{\delta}{\delta J(x)})$ can be written out from Eq. (57) when the field functions in Eq. (57) are replaced by the differentials with respect to the corresponding sources.

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